Advances in Discrete-Time Sliding Mode Control

Theory and Applications

Ahmadreza Argha
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CRC Press
Taylor & Francis Group
Boca Raton London New York

CRC Press is an imprint of the
Taylor & Francis Group, an **informa** business

CRC Press
Taylor & Francis Group
6000 Broken Sound Parkway NW, Suite 300
Boca Raton, FL 33487-2742

Printed on acid-free paper
Version Date: 20180515

International Standard Book Number-13: 978-1-138-30027-9 (Hardback)

Ahmadreza Argha dedicates this book to his wife Naghmeh Akhtar.
Steven W. Su dedicates this book to his family.
Li Li dedicates this book to his family.
Hung T. Nguyen dedicates this book to his family.
Branko G. Celler dedicates this book to his family.

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Symbols

Symbol Description

I_m	Identity matrix of size $m \times m$
A^T	Transpose of the matrix A
$\lambda(A)$	Eigenvalues of the matrix A
$\lambda_{\max}(A)$	Maximum eigenvalue of the matrix A
$\lambda_{\min}(A)$	Minimum eigenvalue of the matrix A
$\mathbb{R}$	Collection of real numbers
$\|x\|$	2-Norm of the vector x defined as $\sqrt{x^T x}$
$\|A\|$	2-Norm of the matrix A defined as max $\frac{\|Ax\|^2}{\|x\|^2}$
$\lvert x(t)\rvert$	Absolute value of the scalar x at time t
$\|x(t)\|$	2-norm of the vector x at time t
$\dot{x}(t)$	Time derivative of the vector $x(t)$, i.e. $\dot{x}(t) = \frac{dx(t)}{dt}$
$[\Sigma_{ij}]_{r\times r}$	is a block matrix with block entries Σ_{ij}, $i = 1,\cdots,r$, $j = 1,\cdots,r$
$\text{diag}\,[\Sigma_{ii}]_{i=1}^{r}$	is a block-diagonal matrix with block entries Σ_{ii}, $i = 1,\cdots,r$
$\text{col}(\nu_i(k))_{i=1}^{r}$	denotes a block-vector with block entries $\nu_i(k)$, $i = 1,\cdots,r$
$\{\circ\}$	denotes an operator for $\Xi = [\xi_{ij}]_{h\times h}$ in which $\xi_{ij} \in \mathbb{R}$ and $W = [W_{ij}]_{h\times h}$ in which $W_{ij} \in \mathbb{R}^{r_i\times s_j}$ such that $\Xi \circ W = [\xi_{ij}W_{ij}]_{h\times h}$
$\otimes$	Kronecker product
SMC	Sliding Mode Control
VSDCS	Variable Structure Discontinuous Control Strategy
CSMC	Continuous-Time Sliding Mode Control
DSMC	Discrete-Time Sliding Mode Control
QSM	Quasi Sliding Mode
LMI	Linear Matrix Inequality
PIO	Proportional Integral Observer
NCS	Networked Control System
MIMO	Multiple Input, Multiple Output
SISO	Single Input, Single Output
OSMC	Output Based Sliding Mode Control
OCSMC	Output Based Continuous-Time Sliding Mode Control
ODSMC	Output Based Discrete-Time Sliding Mode Control
RMS	Root Mean Squares
RLS	Recursive Least Squares
ISMC	Integral Sliding Mode Control
2D	Two-Dimensional
1D	One-Dimensional
FM	Fornasini and Marchesini
RM	Roesser Model

WAM	Wave Advanced Model	HR	Heart Rate
PID	Proportional, Integral and Derivative		

Preface

Sliding mode control (SMC) commenced in the Soviet Union in the late 1950s, but this new control technique was not published until the publications [70] and [113]. Then, the sliding mode research community expanded quickly and the number of publications on this control framework grew correspondingly. Due to the fact that SMC relies on an infinite switching frequency of the input signal, it is inherently a continuous-time control strategy. However, the infinite switching is not achievable in real applications, especially for discrete-time controllers whose input signal can only be varied at the sampling instances. This fact limits the switching frequency to the discrete-time system's sampling frequency. It is worth noting that in a number of applications the assumption of an infinite switching frequency can be relatively justified. In the case that the sampling rate is much faster than the dynamics of the system under control, the influence of the bounded switching frequency will be confined. It is thus a usual approach to design sliding mode controllers in the continuous-time domain, even if the system is computer-aided-controlled [149], regarded as continuous-time sliding mode controller (CSMC), since it is designed according to a continuous-time model of the system, regardless of the sampling issue. However, the effectiveness of the obtained controller will, in addition to many other parameters, strongly depend on the sampling frequency. It means that the faster sampling is performed, the less the influence of the sampling rate will be. More importantly, for a relatively low sampling frequency, the limited switching frequency may result in undesirable effects on the input signal or even instability of the closed-loop system.

Alternatively, the idea of discrete-time sliding mode control (DSMC) has been proposed in literature, which is significantly different from its continuous-time counterpart; see [83] for more information. The results presented in e.g. [83] demonstrate that an appropriate choice of sliding surface, used with the *equivalent control*, can ensure a bounded motion about the surface in the presence of bounded matched uncertainty. Notice also that from this viewpoint, the DSMC problem can be seen as a robust optimal control problem and is related to discrete-time Lyapunov min-max problems [83]. The problem is to select, among all possible feedback controllers, the feedback gain that minimizes the worst case effect of the uncertainty on the Lyapunov difference function [83]. Moreover, the discrete-time equivalent control law can be considered as a solution of the discrete-time linear quadratic regulator (LQR) problem under the assumption of *cheap control*; that is, no penalty is assigned to the control effort in the cost function.

In this book, we explain our recent investigations to improve DSMC and adopt this control strategy to different fields.

The first introductory chapter (Chapter 1) discusses the reasons to consider DSMC. Furthermore, for tutorial purposes, a brief review of CSMC is given in the context of a second-order system. Lastly, in this chapter, the well-known regular form-based method for the design of SMC is reviewed in the framework of discrete-time systems.

Chapter 2 first provides an overview of the relevant literature and places the contribution of the book in a proper context. Further in this chapter, two new forms of switching function are proposed which can be more efficient in terms of reducing the ultimate bound on the system state and reducing the chattering created by traditional switching functions. This new switching function basically uses a disturbance estimator which comes from the same idea presented in [133]. The main idea is, with the assumption of continuity of the original continuous-time disturbance signal, to use the previous value of the sampled disturbance for estimating the current one in the control law. However, model uncertainty is not considered in [133]. In Chapter 2, it is also discussed that using the mentioned estimator directly in the controller will increase the order of the system and, in addition, it results in a system involving time-delay. Stability analysis and ultimate boundedness are then investigated for this kind of system. This method greatly reduces the conservatism of the current linear matrix inequality (LMI)-based methods presented in the few existing works that consider the problem of applying DSMC to the systems including unmatched uncertainties. Specifically, this method avoids using inequalities to deal with the uncertain negative signum quadratic terms appearing in the derived Riccati-like inequality, which is not easy to be directly arranged as an LMI problem. Instead, a *lossless* technique is proposed to convert the mentioned inequality to a form that can be easily written as an LMI. These results were previously published in the paper [13].

While Chapter 2 proposes a state feedback DSMC for uncertain discrete-time systems whose whole states' information is available, Chapter 3 proposes an observer-based output feedback DSMC for discrete-time multi-input multi-output (MIMO) systems. This is more practical, as in many real applications, only systems' output is accessible. Furthermore, the disturbance estimator in Chapter 2 has been designed for the cases that the system states are entirely available. By exploiting output information only for discrete-time MIMO systems with unmatched disturbances and without uncertainties, a framework has been proposed in [32]. Chapter 3 uses an integral term of the estimation output error, in addition to the well-known Luenberger observer which observes the system state with a proportional loop, to allow more degrees of freedom. This matter is referred to as *proportional integral observer* (PIO) in the literature [32]. Nevertheless, the underlying system in [32] does not involve unmatched uncertainties, unlike the system considered in this chapter. The proposed *scheme* here extends the problem of utilizing disturbance observer in the output feedback DSMC (ODSMC) to uncertain discrete-time systems using an innovative LMI based framework. Many of the results in Chapter 3 were previously published in the conference paper [11].

The main goal of Chapter 4 is to stabilize a networked control system (NCS) involving consecutive data packet dropout with a sliding mode control strategy that can improve the existing approaches. In doing so, a novel sliding function is introduced by employing the available communicated system states involving packet losses. This is significantly different from the existing DSMC in the literature [101, 33], and it also provides the possibility to directly build the switching component of the DSMC by exploiting only the available system states. The results in Chapter 4 are based on the papers [6, 15].

The DSMC, given for NCSs in Chapter 4, is derived based on two major assumptions:

1. the packet losses occur only in the channel from the sensor to the controller;
2. the system states are entirely available.

However, these assumptions may be unrealistic for many practical problems. Thus Chapter 5 intends to design sliding mode controllers for NCSs involving both measurement and actuation consecutive packet losses (or long-term random delays), which exploit only output information. This ODSMC can distinguish itself from the existing literature on the SMCs applied to the NCSs, in the sense that both the measurement and actuation delays are viewed as the Bernoulli distributed white sequence. The results in Chapter 5 were previously published in the paper [7].

Decentralized SMC has previously been developed in the literature for large-scale interconnected systems [144, 145, 112, 92]. However, distributed SMC has received less attention and hence it requires more investigation. Chapter 6 first explores the problem of designing a sparse DSMC network for a given plant network with arbitrary topology. To do so, this chapter considers a priori the control network topology which is a subset of the underlying dynamics network and provides a methodology to stabilize the underlying dynamics utilizing a (sparse) distributed observer and controller network. We will show that the proposed observer-based DSMC has the ability to cover all the cases such as decentralized, distributed, and sparsely distributed topologies. In Chapter 6, as the second step, we will search for a sparse control/observer network structure with the least possible number of links that can satisfy the given stability condition. To this end, a heuristic iterative algorithm will be proposed, distinguishing itself from a trial-and-error process which requires checking of all the possible structures. These results were previously published in the conference paper [14].

Although the SMC is now a well-known strategy, from the standpoint of constraining the available control action, all the traditional methods considered in the literature have shortcomings. This drawback basically comes from the nature of the SMC design process which contains two separate stages. During the synthesize of the sliding function, there is no sense of the control action level that is required to induce and retain sliding. This issue is more crucial in Chapter 7 when it comes to sparsifying the control network structure, as without limits on the available control actions, it may result in the high level of control efforts that each subsystem's controller requires to apply, which is not a practical case. Chapter 7 develops an approach by which we can deal with an $\mathscr{H}_2$ based optimal structured SMC problem. In this chapter in order to address the problem of designing a sparse SMC controller, a specific form of fictitious system, whose matrices contain the control network structure, is derived. This makes the well-developed weighted ℓ_1 algorithm infeasible to apply to our problem. Alternatively, Chapter 7 proposes a heuristic scheme to obtain the sparse sliding mode controller. The results in Chapter 7 were published in the papers [12, 8].

According to the so-called 1D quasi-sliding mode, SMC design has been extended for 2D systems in the Roesser Model (RM). In addition, the conditions to ensure the remaining horizontal and vertical states in RM on the switching surfaces and also the reaching condition using a 2D Lyapunov function are investigated in [3]. Another strategy to work with 2D systems is to transfer them to a 1D form. Wave advance model (WAM) is a 1D form of 2D systems established in [111]. From the view point of WAM model, 2D systems are considered as advanced waves and consequently the original stationary 2D system is converted to a time-varying 1D system. Moreover, the system matrices are in rectangular form rather than square form. As a result, the major drawback of this 1D form of 2D systems is the varying dimensions of the defined state vectors. This means that the results developed using this framework are most likely computationally unattractive in terms of possible applications. Motivated by this issue and by the use of stacking vectors, a new approach to converting 2D systems to a 1D form is proposed in Chapter 8. Consequently, the states, inputs and outputs of the obtained 1D system are in the vector form, and more importantly their dimensions are invariant. This framework is basically useful for a class of 2D linear systems in which information propagation in one of the two distinct directions only occurs over a finite horizon. This can be the case of a repetitive process [50] or any inherently 2D system, for instance, the Darboux equation [73]. The suggested 1D vectorial form in Chapter 8 unlike the WAM form has invariable dimension and consequently can be converted to *regular form* in SMC. In this chapter, first the Fornasini and Marchesini (FM) model of 2D systems which is a second order recursive form is considered. The results in Chapter 8 for 2D systems were published in the paper [5].

In Chapter 9, first, the controllability analysis of the WAM model of the first FM model is studied, and a necessary condition for the controllability of this 1D model is given. On the other hand, during the procedure of designing the sliding surface in Chapter 8, it is assumed that the obtained 1D system is controllable. But, the controllability of the obtained 1D form and its relation to the original 2D system is an unanswered problem in Chapter 8. Hence, motivated by these issues, in this chapter, we focus on the controllability analysis of the proposed 1D form of the underlying 2D systems. Based on the controllability analysis, a new notion, *directional controllability*, for the underlying 2D systems is introduced and studied. More importantly, a necessary and sufficient condition for the directional controllability of 2D systems is presented in this chapter. The controllability analyses of 2D systems here were published in the papers [9, 10].

Finally, Chapter 10 is devoted to the problem of heart rate regulation during cycle-ergometer exercise using both a non-model-based as well as a model-based control strategy along with a real-time damped parameter estimation scheme. The model-based control strategy is a time-varying integral sliding mode controller. A recursive damped parameter estimation method is also developed, by incorporation of a weighting upon the one-step parameter variation, which in contrast to the conventional parameter estimation schemes can avoid the occurrence of the so-called blowup phenomena. The calculated control signals are transmitted to the subjects employing a synchronized biofeedback mechanism. Indeed, delivering a feedback signal when the pedals are not in a suitable position to efficiently exert force may be ineffective and this may, in turn, lead to the cognitive disengagement of the user from the feedback controller. Chapter 10 examines a novel form of control system which has been designed for this project. The system is called an "actuator-based event-driven control system". The proposed control and estimation scheme were experimentally verified using several healthy male participants and the results demonstrated that the designed scheme is able to regulate the HR of the exercising subjects to a predetermined HR profile preventing overshooting in the HR responses. The results in this chapter are based on the published papers [16, 17, 18, 19, 20, 21].

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1

Introduction

CONTENTS

Abstract– *Why discrete-time sliding mode control?*
While a large number of investigations in the control systems literature focus on the analysis of continuous-time systems, more and more practising control engineers implement the control laws using digital computers. The controllers can either be carried out from continuous-time representations using fast sampling ideas, or the continuous-time controllers can be converted to their discrete-time representations. However, the choice of the high sampling rate, which nearly approximates continuous-time, may not always be possible. Alternatively, discrete-time controllers can be designed directly from a discrete-time representation of the plant. As a result, one thread of the literature develops discrete-time controllers to stabilize discrete-time linear systems.

In this book, our main focus is on the design of a specific control strategy using digital computers. This control strategy referred to as sliding mode control (SMC) has its roots in (continuous-time) relay control. In fact, as the SMC technique relies on an infinite switching frequency of the input signal, it is inherently a continuous-time control strategy. However, this matter can never be met in real applications, especially for discrete-time controllers where the input signal can only be varied at the sampling instances. This fact can limit the switching frequency to the sampling frequency. Nevertheless, in the case that the sampling rate is much faster than the dynamics of the system under control, the influence of the bounded switching frequency will be confined. It is thus a usual approach to design sliding mode controllers in the continuous-time domain, even if the system is computer-aided-controlled [149], regarded as continuous-time sliding mode controller (CSMC), since it is designed according to a continuous-time model of the system, regardless of the sampling issue. However, the effectiveness of the obtained controller will strongly depend on the sampling frequency, i.e. the faster sampling is performed, the less influence of the sampling rate will be. On the other hand, for a relatively low sampling frequency, the limited switching frequency may result in undesirable effects in the input signal or even instability of the closed-loop system.

This book aims to explain our recent research outcomes in the field of discrete-time sliding mode control (DSMC). The discrete-time systems here are assumed to be obtained by exploiting the sample-and-hold method of sampling from continuous-time systems. In what follows, we present a brief introduction to the concept of continuous-time SMC, and the regular form-based method for the design of SMC, albeit in the context of discrete-time systems.

1.1 Continuous-time SMC

While considering practical control problems, a discrepancy may exist between the actual system and the model used to describe the system behavior; i.e. what is the system output with a specific input. Discrepancies can occur due to exogenous disturbances, unmodeled dynamics, etc. Usually, in model-based control design schemes, this (inaccurate) mathematical model is used for the design of a controller. As a result, controllers should be able to provide a desired performance for the closed-loop system in the presence of disturbances/uncertainties. This task is the main target of the so-called robust control methods. Sliding mode control technique is indeed one of the robust control approaches among many methods proposed and considered in control theory.

Consider the following uncertain linear-time-invariant (LTI) continuous-time system:

$$\dot{x}(t) = Ax(t) + B[u(t) + \xi(x,u,t)], \tag{1.1}$$

where $x \in \mathbb{R}^n$ and $u \in \mathbb{R}^m$ are the state vector and control input vector. The unknown signal $\xi(x,u,t) : \mathbb{R}^n \times \mathbb{R}^m \times \mathbb{R}_+ \to \mathbb{R}^m$ denotes the matched uncertainty in (1.1) whose Euclidean norm is bounded by a known function.

Definition 1.1 *Consider the following system*

$$\dot{x}(t) = Ax(t) + Bu(t) + \tilde{B}\tilde{\xi}(x,u,t). \tag{1.2}$$

The uncertainty $\tilde{\xi}$ in (1.2) is said to be (un)matched uncertainty, if the range space of the input matrix B (does not) contains the range space of $\tilde{B}$ [43].

Without loss of generality, assume that the matrix B has full rank and $m \leq n$. For example, consider a double integrator system, i.e. A and B matrices in (1.1) are as follows

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \; B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}. \tag{1.3}$$

Now let us design a control law for u that asymptotically steers the system states to the origin; i.e. $x = 0$. As the first choice, let us consider $u = Fx$, where $F \in \mathbb{R}^{1\times 2}$ is a feedback gain matrix which can be designed using numerous available

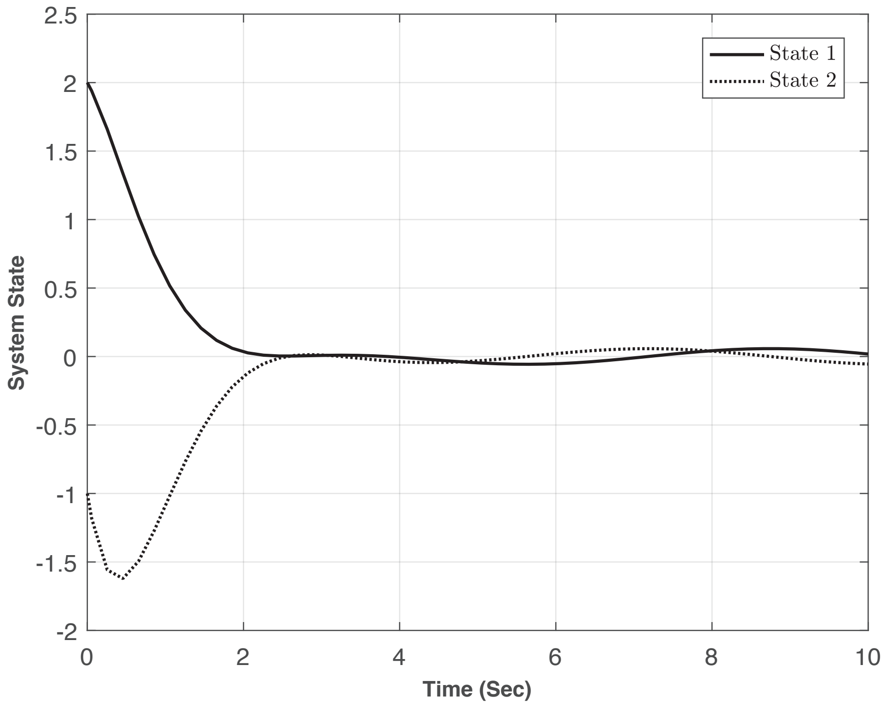

FIGURE 1.1
Evolution of system state using LQ regulator.

approaches. We design F using the linear quadratic regulator (LQR) design approach with $Q = \begin{bmatrix} 10 & 0 \\ 0 & 1 \end{bmatrix}$ and $R = 1$. The obtained gain is $F = \begin{bmatrix} -3.1623 & -2.7064 \end{bmatrix}$, and the poles of the closed-loop system $A + BF$ are located at $-1.3532 \pm 1.1537i$. Fig. 1.1 depicts the evaluation of system states with the proposed LQ regulator when the initial conditions are $x(0) = \begin{bmatrix} 2 & -1 \end{bmatrix}^T$ and $\xi(x,u,t) = 0.2\sin(t)$. As it is evident from Fig. 1.1, this controller cannot asymptotically steer all states to the origin in the presence of ξ. In other words, the LQ regulator can only steer the system states into a region within a bound about $x = 0$. Now, define a new variable σ as

$$\sigma = x_2 - Mx_1, \tag{1.4}$$

where M is a (scalar) design parameter which should be designed such that if $\sigma = 0$ the remaining dynamics are stable. From $\sigma = 0$, we can derive

$$x_2 = Mx_1, \tag{1.5}$$

Substituting (1.5) into $\dot{x}_1 = x_2$, we can obtain $\dot{x}_1 = Mx_1$. This is indeed the dynamics which describes sliding motion. Thus to ensure stability in sliding mode, M

should be a negative scalar. As can be seen from $\dot{x}_1 = Mx_1$, the disturbance ξ has no influence on the sliding mode. From the condition $\dot{\sigma} = \dot{x}_2 - M\dot{x}_1 = 0$, we may obtain

$$\dot{\sigma}(t) = -Mx_2(t) + u(t) + \xi(x,u,t) = 0, \quad \forall t > t_s \tag{1.6}$$

where t_s denotes the time when sliding motion starts. To satisfy $\dot{\sigma} = 0$, a control law can be derived as

$$u_{eq}(t) = Mx_2(t) - \xi(x,u,t). \tag{1.7}$$

This is the so-called equivalent control and is not implementable as ξ is unknown. Indeed, the equivalent control can be regarded as the average control effort required to stay sliding. Now rather than the equivalent control, consider the following control law:

$$u(t) = Mx_2(t) - \rho\,\mathrm{sign}(\sigma(t)). \tag{1.8}$$

It can be shown the above control law can steer σ to zero in finite time if $\rho = \bar{\xi} + \varepsilon$, where $\bar{\xi} > 0$ is a known upper bound on the disturbance ξ, i.e. $\|\xi(x,u,t)\| \leq \bar{\xi}$ and $\varepsilon > 0$ is a small scalar. Consider a candidate Lyapunov function as

$$V = \frac{1}{2}\sigma^2. \tag{1.9}$$

Now,

$$\begin{aligned} \dot{V} &= \sigma\dot{\sigma} = \sigma(Mx_2 - Mx_2 + \xi - \rho\,\mathrm{sign}(\sigma)) \\ &\leq |\sigma|(\bar{\xi} - \rho) = -\varepsilon|\sigma|. \end{aligned} \tag{1.10}$$

This shows the finite-time convergence of the sliding function σ. Note that $\sigma\dot{\sigma} < 0$ is known as reachability condition. Now, we apply the SMC in (1.8), with $M = -1.8875$ and $\rho = 4$, to the system in (1.1) with $\xi(x,u,t) = 0.2\sin(t)$. The results are illustrated in Figs. 1.2-1.6. As it is evident from Figs. 1.2 and 1.3, the SMC in (1.8) ensures the finite-time convergence of the sliding function as well as asymptotic convergence of system states to zero when $\xi \neq 0$. The reaching phase and the sliding phase can be seen in Fig. 1.6. However, as can be seen in Figs. 1.4 and 1.5, due to the practical limitations on the sign function implementation, the so-called chattering phenomenon occurs while using the SMC (1.8). It is worth noting that in some applications such a switching is inherent, e.g. electrical converters. However, broadly speaking, in many other applications the high frequency switching is undesirable [98].

Since the actuator bandwidth is usually limited, an infinite switching frequency is not achievable. Also, the high frequency control signals in real applications may have harmful consequences, e.g. large current peaks in electrical actuators and high wear in mechanical gear boxes. One simple and useful method to make the discontinuous component in (1.8) continuous and smooth is approximating sign$(\cdot)$ by some continuous/smooth function. For example, sigmoid function is a well-known choice [43]:

$$u_n = -\rho\frac{\sigma}{|\sigma| + \varepsilon}, \tag{1.11}$$

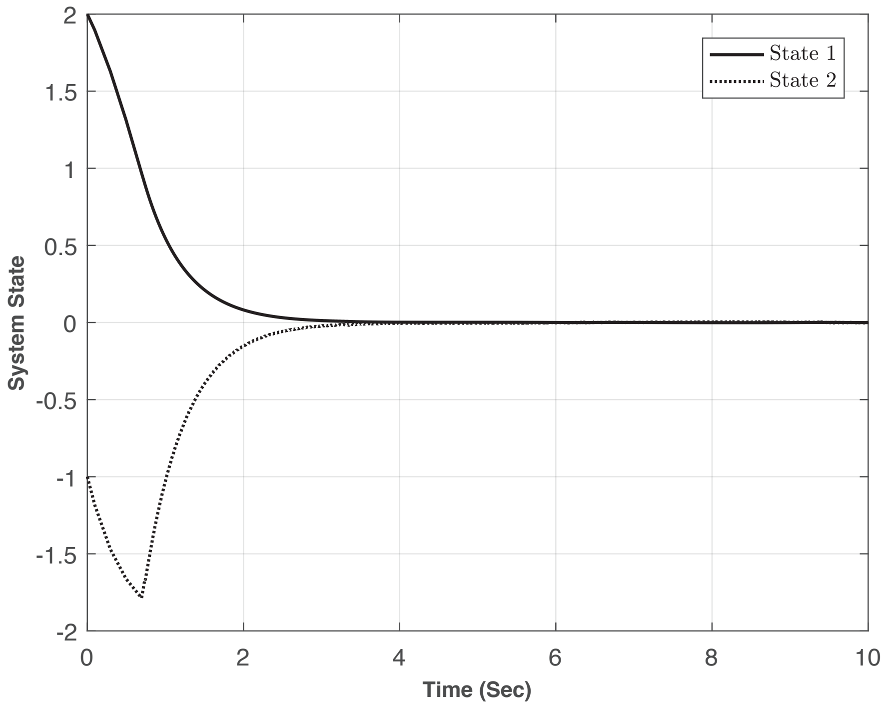

FIGURE 1.2
Evolution of system state obtained by applying SMC in (1.8) with $M = -1.8875$ and $\rho = 4$ to the system 1.1.

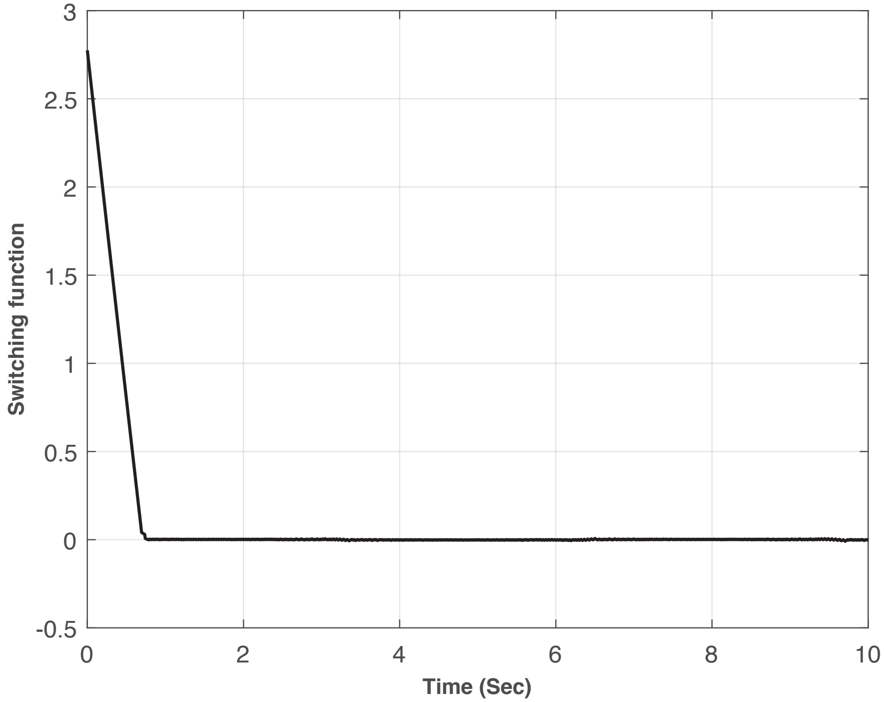

FIGURE 1.3
Evolution of switching function for the system 1.1 using SMC in (1.8) with $M = -1.8875$ and $\rho = 4$.

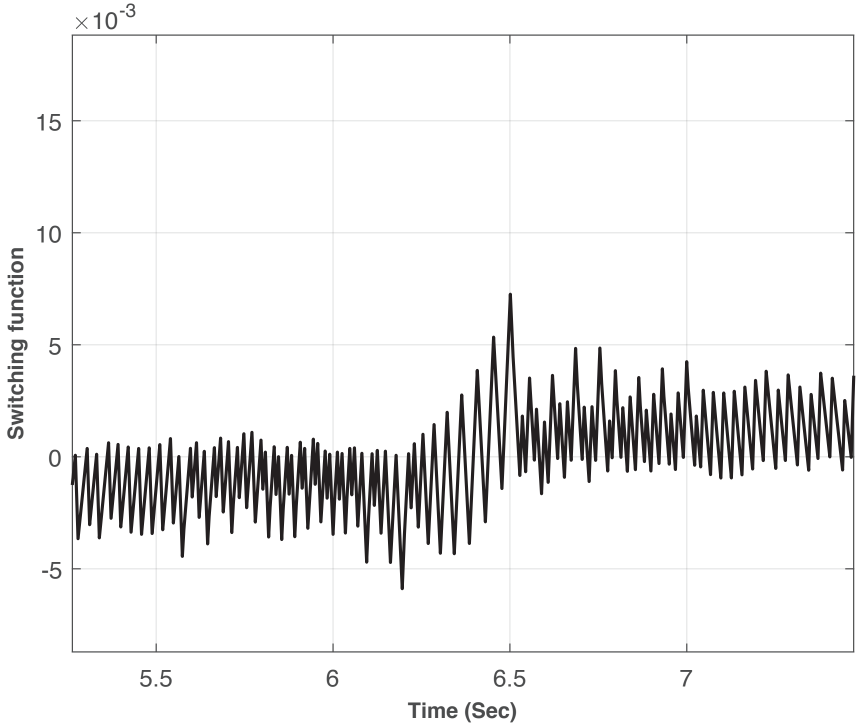

FIGURE 1.4
Evolution of switching function (zoom) for the system 1.1 using SMC in (1.8) with $M = -1.8875$ and $\rho = 4$.

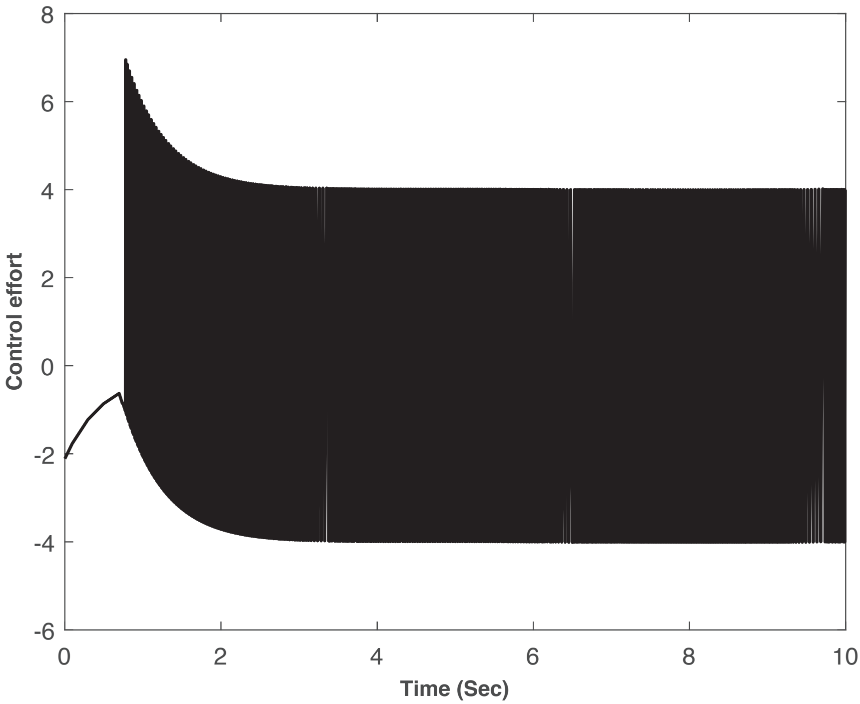

FIGURE 1.5
Control effort with SMC in (1.8) with $M = -1.8875$ and $\rho = 4$.

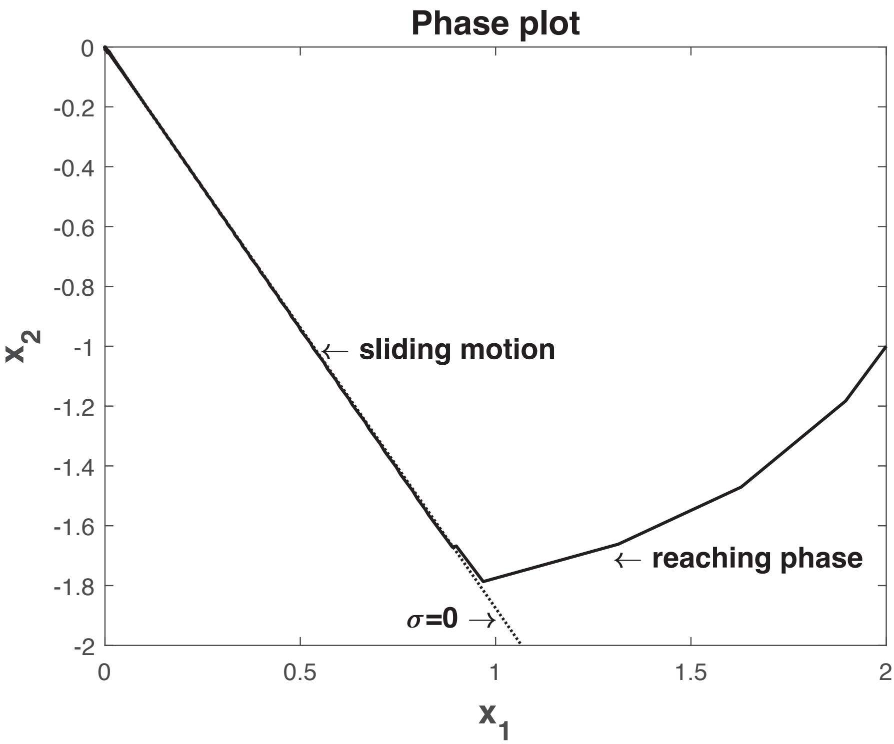

FIGURE 1.6
Phase plot for the system (1.1) using SMC in (1.8) with $M = -1.8875$ and $\rho = 4$.

where $\varepsilon > 0$ is a small scalar and u_n denotes the nonlinear controller part of the SMC (1.8). Note that ε is a design freedom to trade off between having an ideal performance and ensuring a smooth control signal. Let us replace $\rho\text{sign}(\sigma)$ with $\rho\frac{\sigma}{|\sigma|+\varepsilon}$ in (1.8) to yield:

$$u(t) = Mx_2(t) - \rho\frac{\sigma}{|\sigma|+\varepsilon}. \tag{1.12}$$

Applying this new controller, with $M = -1.8875$, $\rho = 4$ and $\varepsilon = 0.1$, to the system (1.1) leads to the results shown in Figs. 1.7-1.9. As it is evident from these results, the controller (1.12) does not lead the switching function σ to converge to the origin in finite-time when $\xi \neq 0$, and further the system states do not converge to zero. However, the sliding variable converges to a bound around $\sigma = 0$ and the system states converge to a region within a bound about $x = 0$. The controller (1.12) is referred to as *quasi sliding mode control* and the boundary region about $\sigma = 0$ described previously is called *quasi sliding mode band*.

Now a more practical controller rather than (1.12) can be proposed as

$$u(t) = Mx_2(t) + \varphi\sigma - \rho\frac{\sigma}{|\sigma|+\varepsilon}, \tag{1.13}$$

where $\varphi < 0$ is a scalar which can be used along with ρ to change the convergence rate of sliding variable to a bound around $\sigma = 0$. Note that by the choice of sliding surface (1.4), it follows from (1.6) that

$$\dot{\sigma}(t) = \varphi\sigma - \rho\frac{\sigma}{|\sigma|+\varepsilon} + \xi(x,u,t). \tag{1.14}$$

Let us analyze the reachability of the bound $|\sigma| \geq \delta$, where $\delta = \frac{\bar{\xi}\varepsilon}{\rho-\bar{\xi}}$, with the new controller (1.13). We can consider the reachability condition

$$\sigma\dot{\sigma} \leq -\eta|\sigma|, \tag{1.15}$$

where $\eta > 0$ is a small scalar. Now it follows from (1.15) and (1.14) that

$$\begin{aligned} \sigma\dot{\sigma} &= \sigma\left(\varphi\sigma - \rho\frac{\sigma}{|\sigma|+\varepsilon} + \xi\right) \\ &\leq |\sigma|\left(\varphi|\sigma| - \rho\frac{|\sigma|}{|\sigma|+\varepsilon} + \bar{\xi}\right) \\ &\leq |\sigma|\left(\bar{\xi} - \rho\frac{|\sigma|}{|\sigma|+\varepsilon}\right) \\ &\leq -\eta|\sigma|. \end{aligned} \tag{1.16}$$

The above inequality leads us to

$$|\sigma| \geq \frac{(\eta+\bar{\xi})\varepsilon}{\rho-\eta-\bar{\xi}}. \tag{1.17}$$

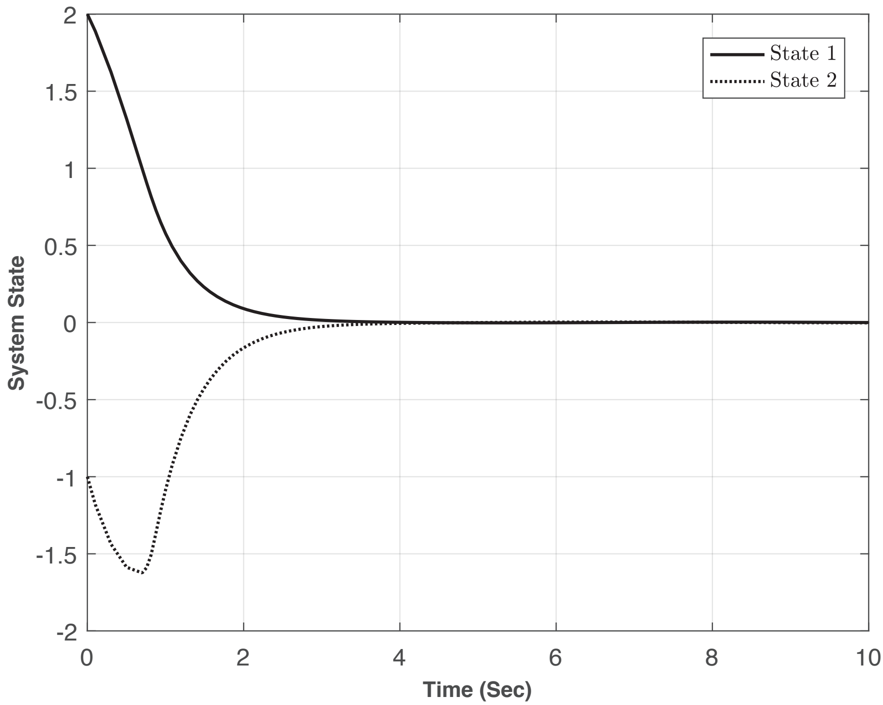

FIGURE 1.7
Evolution of system state obtained by applying SMC in (1.12) with $M = -1.8875$, $\rho = 4$ and $\varepsilon = 0.1$ to the system (1.1).

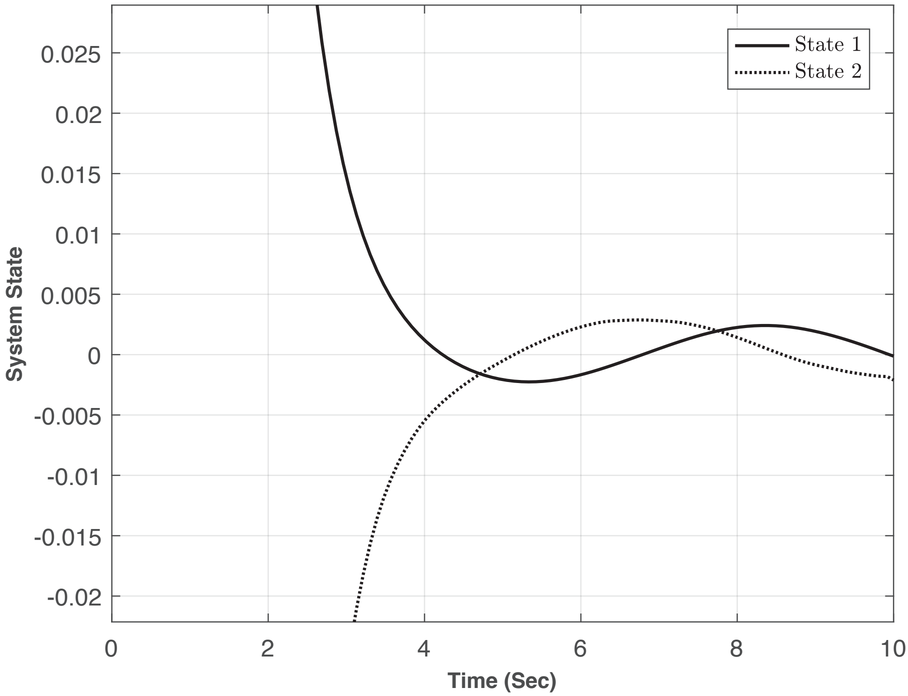

FIGURE 1.8
Evolution of system state (zoom) obtained by applying SMC in (1.12) with $M = -1.8875$, $\rho = 4$ and $\varepsilon = 0.1$ to the system (1.1).

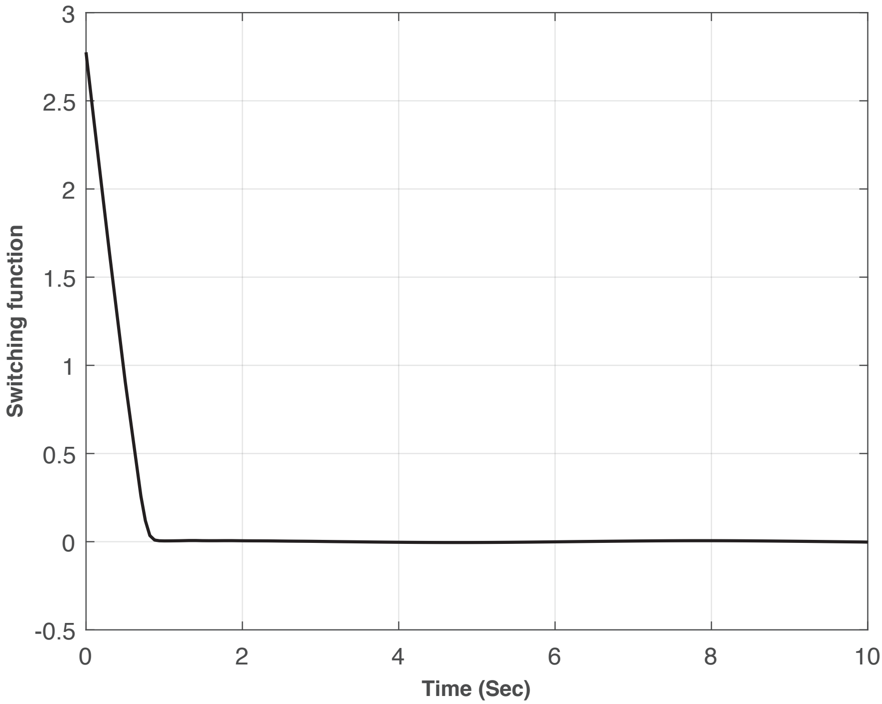

FIGURE 1.9
Evolution of switching function for the system (1.1) using SMC in (1.12) with $M = -1.8875$, $\rho = 4$ and $\varepsilon = 0.1$.

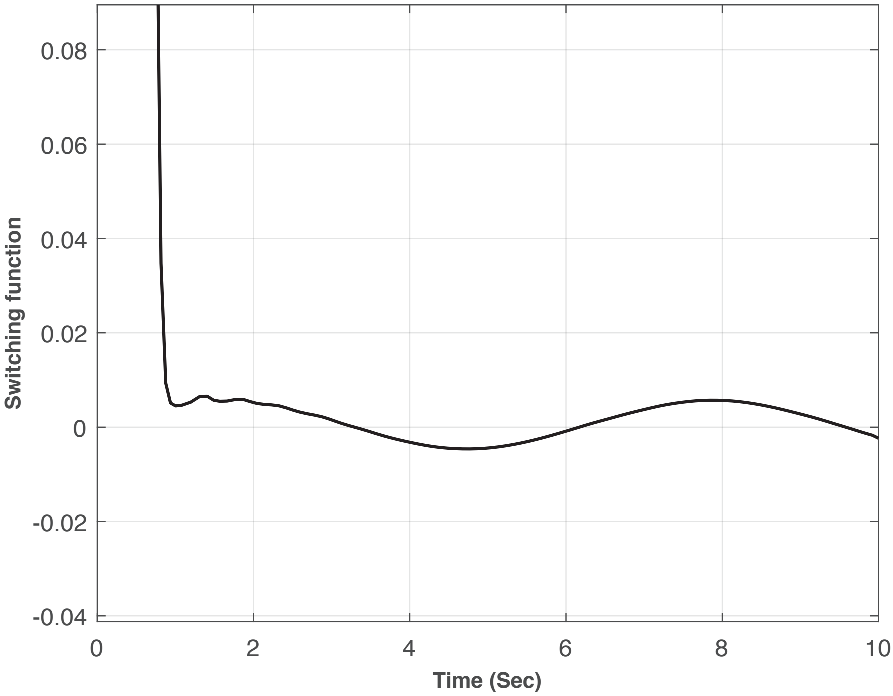

FIGURE 1.10
Evolution of switching function (zoom) for the system (1.1) using SMC in (1.12) with $M = -1.8875$, $\rho = 4$ and $\varepsilon = 0.1$.

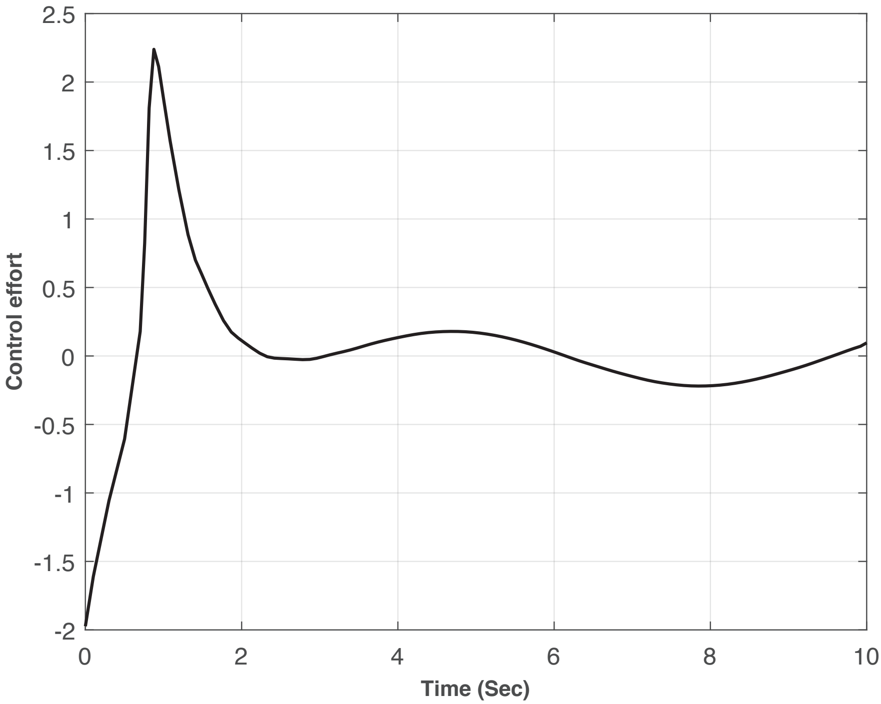

FIGURE 1.11
Control effort with SMC in (1.12) with $M = -1.8875$, $\rho = 4$ and $\varepsilon = 0.1$.

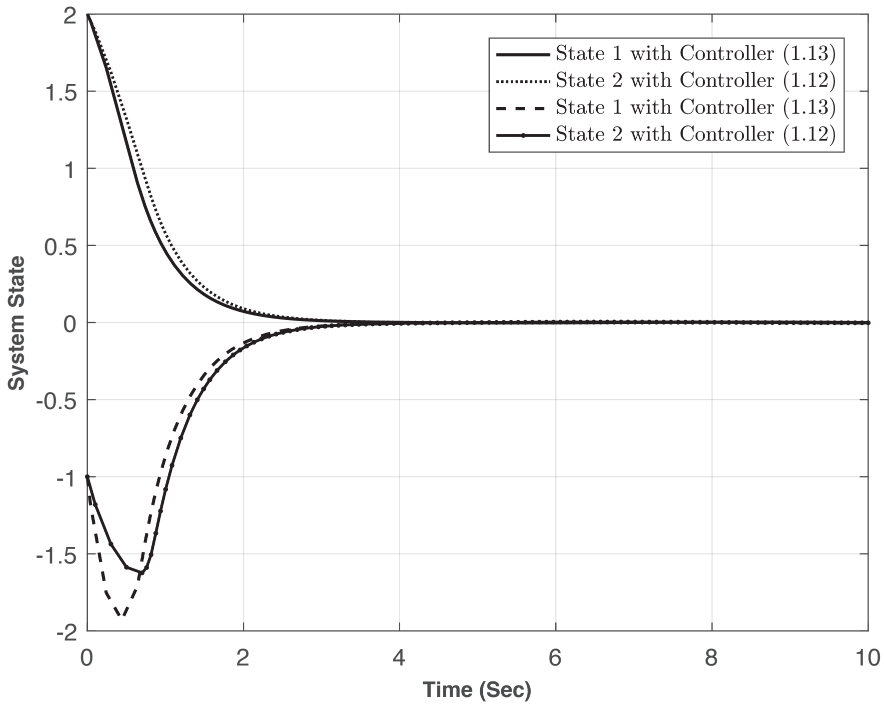

FIGURE 1.12
Evolution of system state obtained by applying SMC in (1.13) and (1.12) with $M = -1.8875$, $\rho = 4$, $\varepsilon = 0.1$ and $\varphi = -1$ to the system (1.1).

By taking the scalar $\eta > 0$ very small, the above given bound on $|\sigma|$ reduces to

$$|\sigma| \geq \frac{\bar{\xi}\varepsilon}{\rho - \bar{\xi}}. \tag{1.18}$$

In summary, if $|\sigma| \geq \frac{\bar{\xi}\varepsilon}{\rho-\bar{\xi}}$, the controller (1.13) will force the system states into the quasi SMC band $\delta = \frac{\bar{\xi}\varepsilon}{\rho-\bar{\xi}}$.

With the same choice of $M = -1.8875$, as used previously, and letting $\varphi = -1$, $\rho = 4$ and $\varepsilon = 0.1$, we apply the controller (1.13) to the system (1.1) and the obtained results are shown in Figs. 1.12-1.15.

As it is evident from Figs. 1.12-1.15 the new parameter φ in the controller can be used, along with the parameter ρ, to set the convergence rate to the sliding surface (or indeed into the quasi sliding band). Quasi sliding is obtained in 0.64 s with the controller (1.13), while it takes around 0.94 s for the controller (1.12) to drive the sliding variable to the quasi sliding band.

The double integrator system considered here is a single-input system, and the

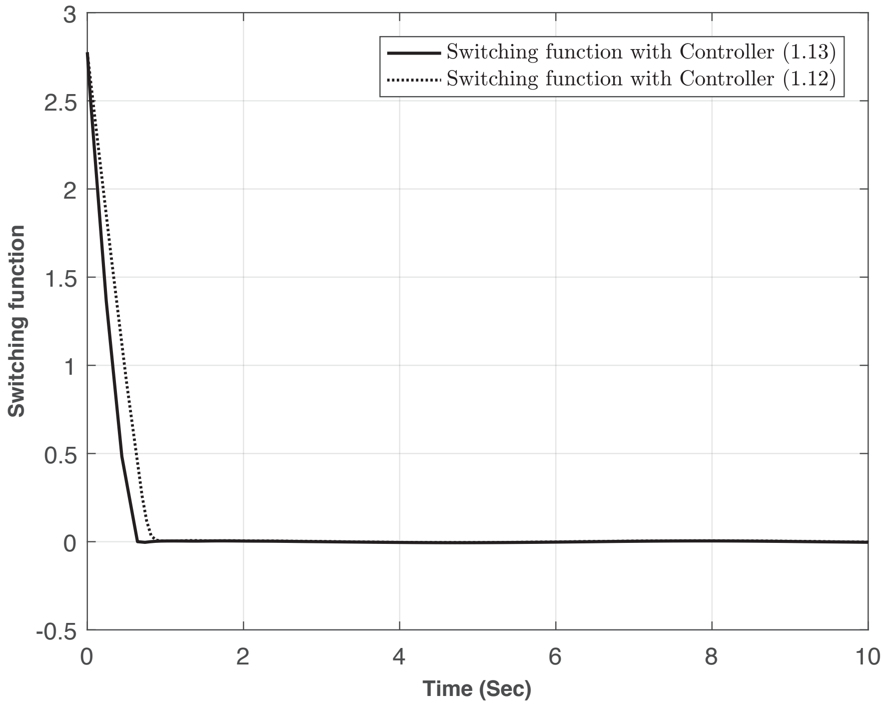

FIGURE 1.13
Evolution of switching function for the system (1.1) using SMC in (1.13) and (1.12) with $M = -1.8875$, $\rho = 4$, $\varepsilon = 0.1$ and $\varphi = -1$.

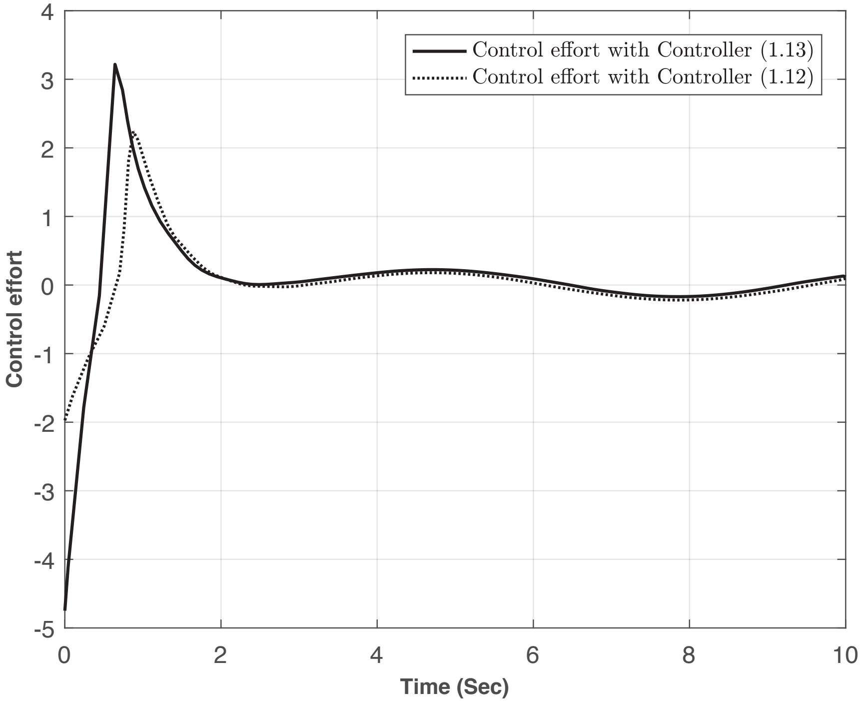

FIGURE 1.14
Control effort with SMC in (1.13) and (1.12) with $M = -1.8875$, $\rho = 4$, $\varepsilon = 0.1$ and $\varphi = -1$.

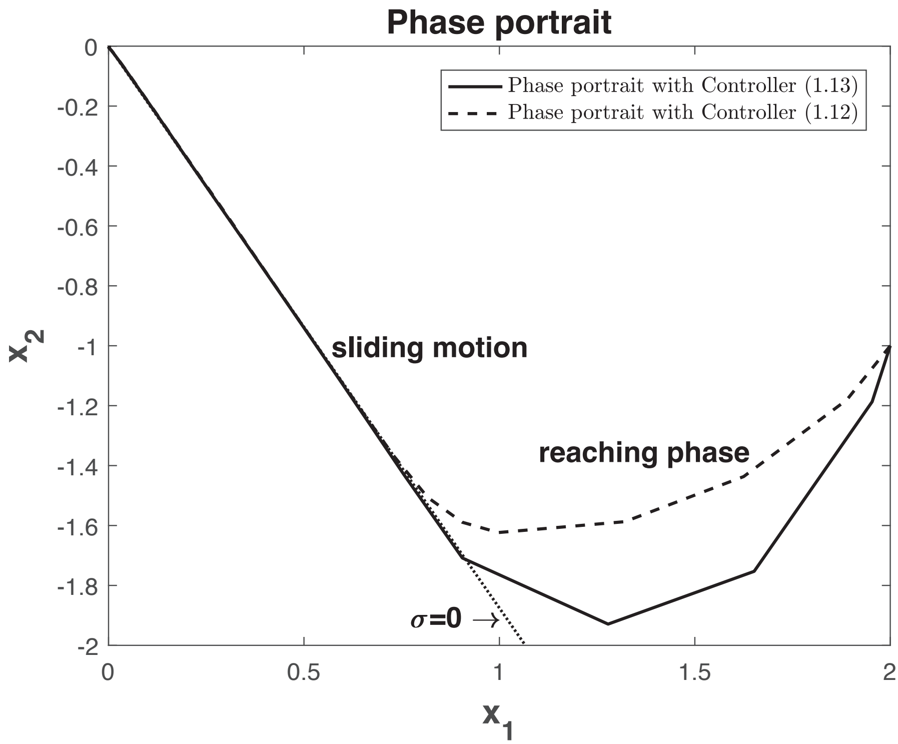

FIGURE 1.15
Phase portrait for the system (1.1) using SMC in (1.13) and (1.12) with $M = -1.8875$, $\rho = 4$, $\varepsilon = 0.1$ and $\varphi = -1$.

sliding mode controller design given above is not extendable to multi-input systems. For multi-input systems, SMC design can be carried out in different ways including the *regular form-based* method. While we give a brief introduction to this method in this chapter, albeit in the context of discrete-time systems, it should be emphasized that the linear matrix inequality (LMI)-based schemes given in the remainder of this book for the design of DSMC is not basically based on the regular form-based SMC design. We only use the regular form-based approach in the design of DSMC for two-dimensional systems in Chapter 8.

1.2 Regular form-based DSMC

Similar to CSMC, the design procedure of the DSMC for stabilizing problems is split into two steps:

1. Design a sliding surface which lead to stable internal dynamics during sliding.
2. Create a control law which drives the closed-loop system into the sliding surface and forces the system trajectories to stay on or at least as close as possible to the surface.

A number of different methods for designing the sliding surface are considered in the literature [43]. In this section, we give a brief introduction to the well-known regular form based approach [43].

Consider the following uncertain linear discrete-time system,

$$x(k+1) = Ax(k) + Bu(k) + f(k), \tag{1.19}$$

where $x(k) \in \mathbb{R}^n$ and $u(k) \in \mathbb{R}^m$. Generally, it is assumed that $B \in \mathbb{R}^{n\times m}$ and $m \leq n$. Besides, $\text{rank}(B) = m$ (matrix B has full column rank) and it is assumed that the pair (A,B) is controllable. Also, $f(k) \in \mathbb{R}^n$ denotes the uncertainty. We consider the following general uncertainty

$$f(k) = \Delta x(k) + d_k, \tag{1.20}$$

where Δ shows the unknown uncertainty with the bound $\|\Delta\| < \alpha_0$ ($\|.\|$ the induced Euclidean or induced spectral norm). Moreover, the term $d_k \in \mathbb{R}^n$, indicates the external disturbance and it is assumed that $\|d_k\| < \beta_0$, where β_0 is a known positive constant. As a result, we can write

$$\|f(k)\| < \alpha_0 \|x(k)\| + \beta_0. \tag{1.21}$$

Since $\text{rank}(B) = m$, there exists an orthogonal matrix $T_r \in \mathbb{R}^{n\times n}$ such that

$$T_r B = \begin{bmatrix} 0_{(n-m)\times m} \\ \bar{B}_2 \end{bmatrix}, \tag{1.22}$$

where the matrix $\bar{B}_2 \in \mathbb{R}^{m\times m}$ is nonsingular [127]. After the coordinate transformation, the system (1.19) is converted to

$$\begin{bmatrix} \bar{x}_1(k+1) \\ \bar{x}_2(k+1) \end{bmatrix} = \begin{bmatrix} \bar{A}_{11} & \bar{A}_{12} \\ \bar{A}_{21} & \bar{A}_{22} \end{bmatrix} \begin{bmatrix} \bar{x}_1(k) \\ \bar{x}_2(k) \end{bmatrix} + \begin{bmatrix} 0_{(n-m)\times m} \\ \bar{B}_2 \end{bmatrix} u(k) + T_r f(k). \tag{1.23}$$

Now, the sliding surface is introduced as

$$\sigma_x(k) = \bar{S}\bar{x}(k) = \bar{S}_1\bar{x}_1(k) + \bar{S}_2\bar{x}_2(k), \tag{1.24}$$

where $\bar{S}_1 \in \mathbb{R}^{m\times(n-m)}$ and $\bar{S}_2 \in \mathbb{R}^{m\times m}$ are the design parameters which determine the sliding surface and should be chosen such that, in the case that $\sigma_x(k) = 0$, all remaining dynamics are stable. During ideal sliding on the surface, $\sigma_x(k) = 0$ for all $k \geq k_s$, where k_s is the time when sliding starts, therefore

$$\bar{x}_2(k) = -\bar{S}_2^{-1}\bar{S}_1\bar{x}_1(k). \tag{1.25}$$

Substituting the equation (1.25) into the equation (1.23) and ignoring the uncertainty $f(k)$ leads to

$$\bar{x}_1(k+1) = (\bar{A}_{11} - \bar{A}_{12}\bar{S}_2^{-1}\bar{S}_1)\bar{x}_1(k). \tag{1.26}$$

Hence, stability in the sliding mode is satisfied when all eigenvalues of the matrix $(\bar{A}_{11} - \bar{A}_{12}\bar{S}_2^{-1}\bar{S}_1)$ are located inside the unit circle. The sliding surface in the original coordinate can be found by $\sigma_x(k) = Sx(k)$, where

$$S = [\bar{S}_1 \;\; \bar{S}_2]T_r. \tag{1.27}$$

Define T_s as

$$T_s = \begin{bmatrix} I_{(n-m)} & 0_{(n-m)\times m} \\ \bar{S}_1 & \bar{S}_2 \end{bmatrix}, \tag{1.28}$$

and in the new coordinate $T_s\bar{x} \longmapsto \tilde{x}$, we have

$$\tilde{x}(k+1) = \begin{bmatrix} \bar{x}_1(k+1) \\ \sigma_x(k+1) \end{bmatrix} = \begin{bmatrix} \tilde{A}_{11} & \tilde{A}_{12} \\ \tilde{A}_{21} & \tilde{A}_{22} \end{bmatrix} \begin{bmatrix} \bar{x}_1(k) \\ \sigma_x(k) \end{bmatrix} + \begin{bmatrix} 0_{(n-m)\times m} \\ \bar{S}_2\bar{B}_2 \end{bmatrix} u(k) + T_sT_r f(k). \tag{1.29}$$

Now, let

$$u(k) = u_l(k) + u_n(k), \tag{1.30}$$

where u_l denotes the linear controller and u_n is the nonlinear component of the DSMC. While we let $u_n = 0$ here, different choices for nonlinear controller in (1.30) will be proposed and discussed later in Chapter 2. Now, consider the following well-known linear sliding control law:

$$u(k) = \left(\bar{S}_2\bar{B}_2\right)^{-1}\left[\left(\Phi - \tilde{A}_{22}\right)\sigma_x(k) - \tilde{A}_{21}\bar{x}_1(k)\right], \tag{1.31}$$

where $\Phi \in \mathbb{R}^{m\times m}$ is a diagonal matrix whose diagonal elements, $\phi_r,\; r = 1,\ldots,m$, satisfy $0 \leq \phi_r < 1$. Thus, with the control law (1.31) the closed-loop system is

$$\tilde{x}(k+1) = \begin{bmatrix} \bar{x}_1(k+1) \\ \sigma_x(k+1) \end{bmatrix} = \begin{bmatrix} \tilde{A}_{11} & \tilde{A}_{12} \\ 0 & \Phi \end{bmatrix} \begin{bmatrix} \bar{x}_1(k) \\ \sigma_x(k) \end{bmatrix} + \begin{bmatrix} \tilde{f}_1(k) \\ \tilde{f}_2(k) \end{bmatrix}. \tag{1.32}$$

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