Controller Design:
Practical System Identification and Digital Transform

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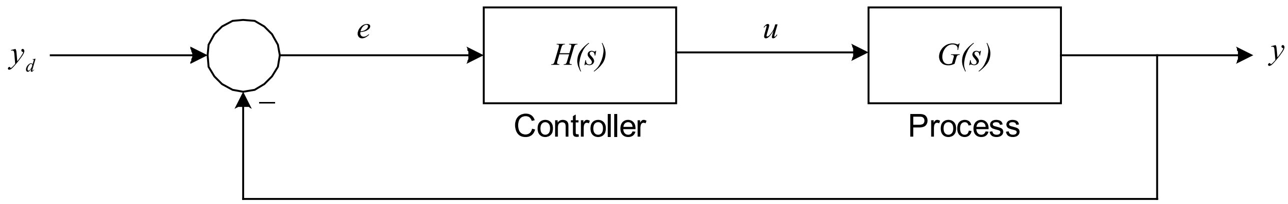
1. Introduction

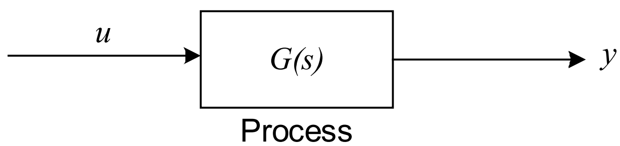
The objective of control job is to get a desired output



where
- y is actual output of the process
- y_d is desired output
- e is error to be eliminated
- u is control effort for the job

The followings are required for designing a controller
- Recognize input and output of the process
- Modeling the process (system identify)
- Design controller using the model for simulation ensure the control effort is within process limit; for example maximum control input to a 12-v servo motor is 12 V.

2. Practical System Identification



We can get the model of the process using step response. Practically we can get 1st and 2nd order at most even for higher order process as we see in PD Controller for Higher Order Systems. We'll, therefore, look at 1st and 2nd order system identification. In reality, a complex system is composed of many blocks and we identify these blocks.

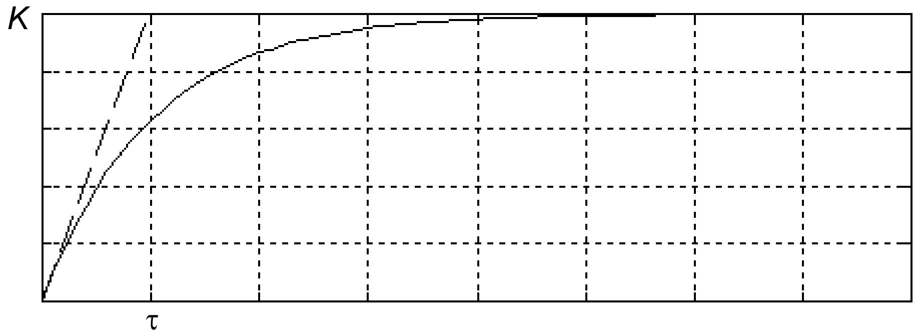
2.1. First Order Systems

Consider the following second order system

$$G(s) = \frac{K}{\tau\omega_n + 1} \tag{1}$$

where τ is time-contant of the system.

The step response is



2.2. Second Order Systems

2.2.1 System 1

2.2.1.1 Analysis

Consider the following second order system

$$G(s) = \frac{K}{s^2 + 2\zeta\omega_n + \omega_n^2} \tag{2}$$

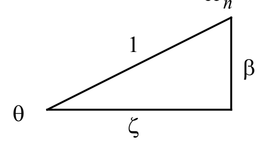
then a step response is

$$Y(s) = \frac{a}{s}\frac{K}{s^2 + 2\zeta\omega_n + \omega_n^2}$$

so

$$y(t) = \frac{aK}{\omega_n^2}\left[1 - \frac{1}{\beta}e^{-\zeta\omega_n t}\sin(\omega t + \theta)\right], \omega = \beta\omega_n \tag{3}$$

$$y_0 = 0, y_{ss} = \frac{aK}{\omega_n^2}$$

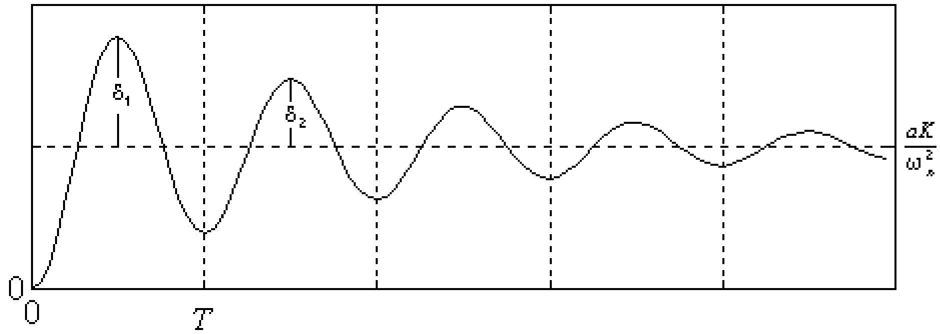


$$T_p = \frac{T}{2} = \frac{\pi}{\omega} = \frac{\pi}{\beta\omega_n}$$

$$\delta_* = \frac{\delta}{y_{ss}} = \frac{1}{\beta}e^{-\zeta\omega_n T_p}\sin(\theta) = e^{-\zeta\omega_n T_p} \Rightarrow -\zeta\omega_n T_p = \ln(\delta_*) \Rightarrow -\zeta\frac{\pi}{\beta} = \ln(\delta_*) \Rightarrow \pi^2\zeta^2 = (1-\zeta^2)\ln^2(\delta_*)$$

$$\zeta = \frac{1}{\sqrt{1 + (\pi/\ln\delta_*)^2}}$$

2.2.1.2 Identification



From the step response graph, we have

$$\delta_* = \frac{\delta_1}{y_{ss}} \Rightarrow \zeta = \frac{1}{\sqrt{1+\left(\pi/\ln\delta_*\right)^2}} \textbf{ or } \delta_* = \frac{\delta_1}{\delta_2} \Rightarrow \zeta = \frac{1}{\sqrt{1+\left(2\pi/\ln\delta_*\right)^2}}$$

$$\omega = \frac{2\pi}{T} \Rightarrow \omega_n = \frac{\omega}{\beta} = \frac{\omega}{\sqrt{1-\zeta^2}}$$

$$K = \frac{y_{ss}\omega_n^2}{a}$$

2.2.2 System 2

2.2.2.1 Analysis

Consider the following second order system

$$G(s) = \frac{Ks}{s^2 + 2\zeta\omega_n + \omega_n^2} \tag{4}$$

then a step response is

$$Y(s) = \frac{aK}{s^2 + 2\zeta\omega_n + \omega_n^2}$$

so

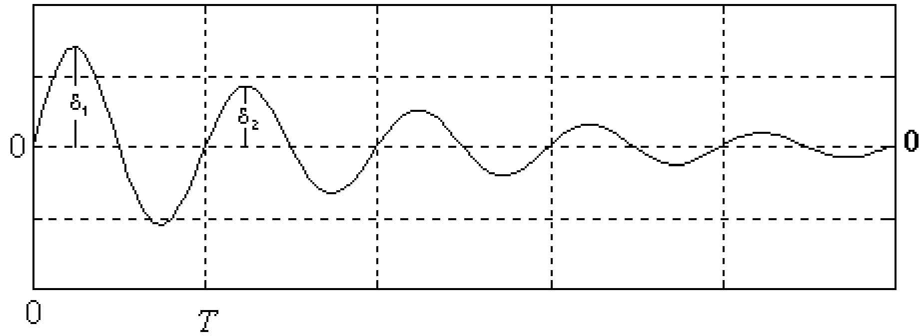
$$y(t) = \frac{aK}{\omega} e^{-\zeta\omega_n t} \sin(\omega t), \omega = \beta\omega_n \tag{5}$$

$$y_0 = 0, y_{ss} = 0$$

$$\omega = \frac{2\pi}{T} \Rightarrow \omega T = \beta\omega_n T = 2\pi$$

$$\zeta = \frac{1}{\sqrt{1+\left(2\pi/\ln\delta_*\right)^2}}$$

2.2.2.2 Identification



From the step response graph, we have

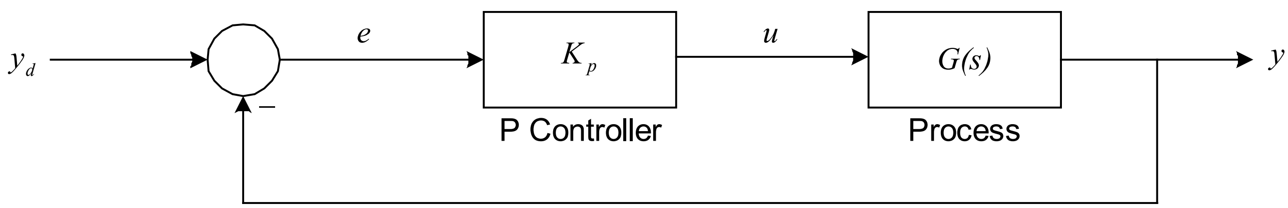
$$\delta_* = \frac{\delta_1}{\delta_2} \Rightarrow \zeta = \frac{1}{\sqrt{1 + \left(2\pi / \ln \delta_*\right)^2}}$$

$$\omega = \frac{2\pi}{T} \Rightarrow \omega_n = \frac{\omega}{\beta} = \frac{\omega}{\sqrt{1-\zeta^2}}$$

3. Controller Design

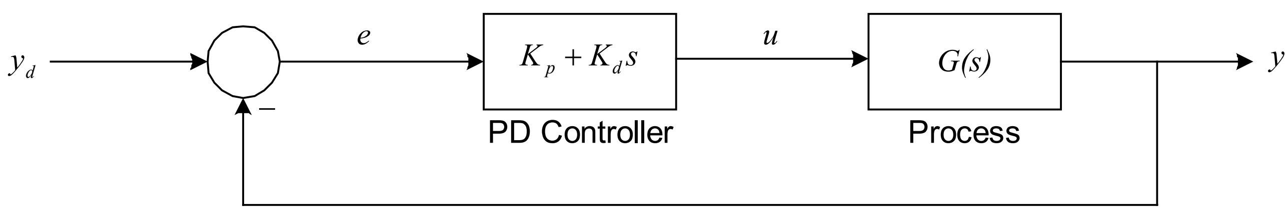
3.1. P Controller (Proportional)

We first start with proportional controller with only 1 control parameter, gain K



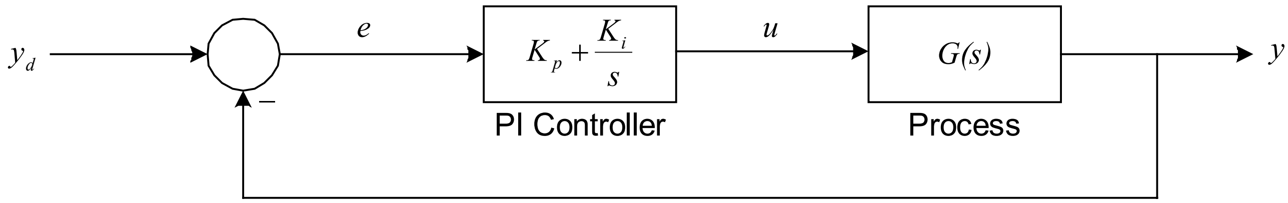
3.2. PD Controller (P with Derivative action)

The limitation of P controller is all we can do is to use higher gain for faster response. However, it may cause over-shoot for 2^{nd} order system. So a D (derivation) action is introduced to account for this problem. The more over-shoot results in higher D action control.

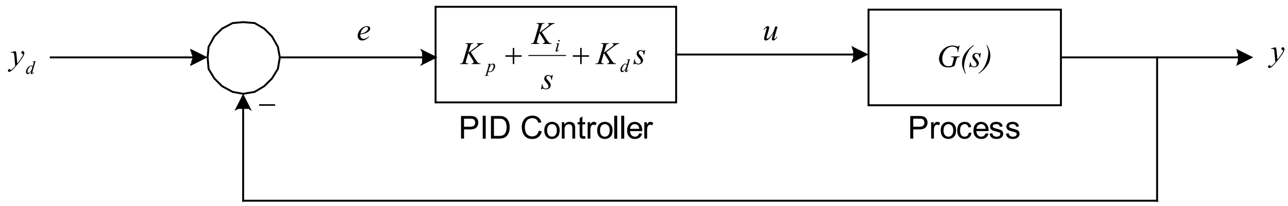


3.3. PI Controller (P with Integration action)

In some system, a P controller cannot produce control effort strong enough to eliminate small error. An I (integration) action is employed to get a large accumulated error, and hence a large control effort.



3.4. PID Controller



4. Practical Digital Controller

A precise presentation of digital control system can be found in Digital Control System. In practice, we can use calculus background to have a practical transform of an analog controller to a digital one

$$\frac{du}{dt} \approx \frac{\Delta u}{\Delta t}, \qquad \int u(t)dt = \sum u(t)\Delta t$$

The key point is we never get an accurate model and we have to do fine-tuning our controller eventually. So it's reasonable to use practical system identification and digital transform above and a fine-tuning controller at last.

P Controller

$$u(t) = K_p e(t) \;\Rightarrow\; u(k) = K_p e(k)$$

PD Controller

$$u(t) = K_p e(t) + K_d \frac{de}{dt} \;\Rightarrow\; u_k = K_p e_k + K_d \frac{e_{k+1} - e_k}{T_s}$$

PI Controller

$$u(t) = K_p e(t) + K_i \int e(t).dt \;\Rightarrow\; u_k = K_p e_k + K_i \sum_k e_k T_s$$

PID Controller

$$u(t) = K_p e(t) + K_i \int e(t).dt + K_d \frac{de}{dt} \;\Rightarrow\; u_k = K_p e_k + K_i \sum_k e_k T_s + K_d \frac{e_{k+1} - e_k}{T_s}$$

where T_s is sampling time.