Control Laboratory Projects

Duy Ky Nguyen **09-02-95**

State-Space Estimator-Controller Design

State-Space Controller Design

$$\begin{cases} \dot{\mathbf{x}} = \mathbf{A}.\mathbf{x} + \mathbf{B}.u, \quad u = -\mathbf{K}.\mathbf{x} \\ \mathbf{K} = \text{place}(\mathbf{A}, \mathbf{B}, \mathbf{P}_c), \quad \mathbf{P}_c: \text{ controller - eigenvalues} \end{cases}$$

State-Space Estimator Design

$$\mathbf{L} = \text{place}(\mathbf{A}^T, \mathbf{C}^T, \mathbf{P}_e)^T, \quad \mathbf{P}_e = (3 \sim 10)\mathbf{P}_c: \text{ estimator - eigenvalues}, \quad \mathbf{C}: \underline{\text{available outputs}}$$

$$\begin{cases} \dot{\hat{\mathbf{x}}} = (\mathbf{A} - \mathbf{LC})\hat{\mathbf{x}} + \mathbf{B}.u + \mathbf{L}.\mathbf{y} \\ \mathbf{u} = -\mathbf{K}.\hat{\mathbf{x}}; \end{cases}$$

Derivation of Time-Constant

If a velocity response is available then the time-constant is determined at 63% of the steady-state value. This section is applicable if such response is not available, a tacho-generator is out of order or does not exist. For a step input of a-amplitude

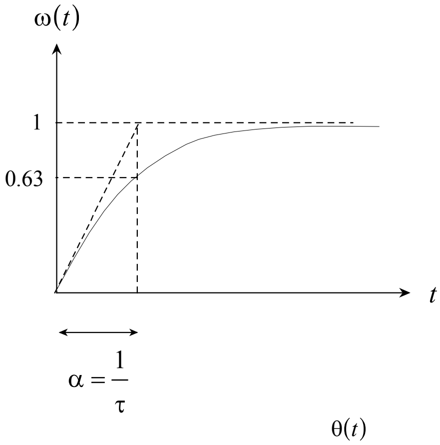
$$\hat{\theta}(s) = \frac{aK_m}{s^2(s+\alpha)}, \quad \alpha = \frac{1}{\tau}$$

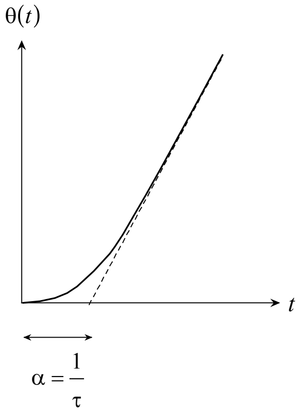
or

$$\theta(t) = aK_m \int_0^t \frac{1}{\alpha}\left(1 - e^{-\alpha u}\right) du = \frac{aK_m}{\alpha^2}\left(\alpha t - 1 + e^{-\alpha t}\right)$$

so its assymptote is

$$\varepsilon(t) = \frac{aK_m}{\alpha^2}(\alpha t - 1)$$

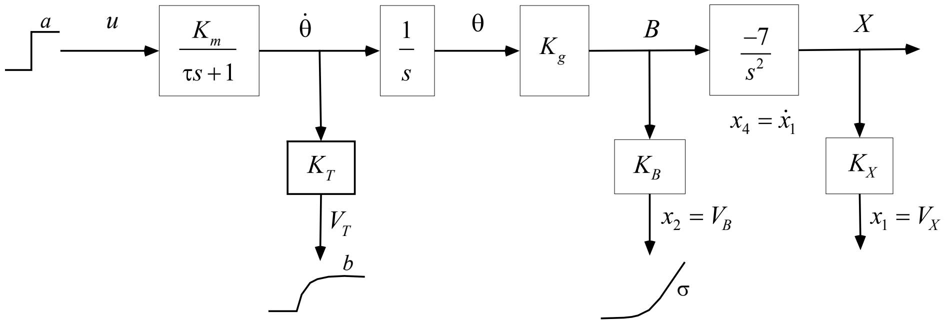

ω(t)
1
0.63
t
α = 1/τ


θ(t)
t
α = 1/τ

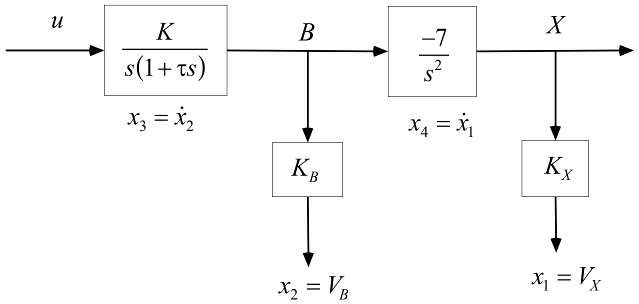
BALL- BEAM SERVO-MOTOR SYSTEM

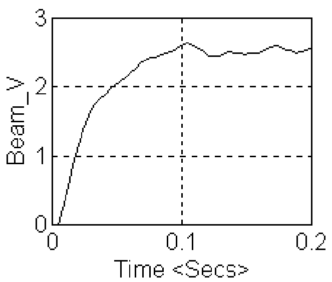
Duy-Ky Nguyen

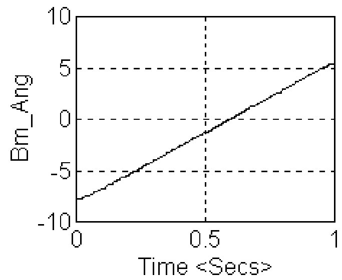
13-11-1993



$$\frac{b}{a} = K_T K_m, \quad \frac{\sigma}{a} = K_B K_g K_m, \quad \frac{V_B}{\theta} = K_B K_g$$



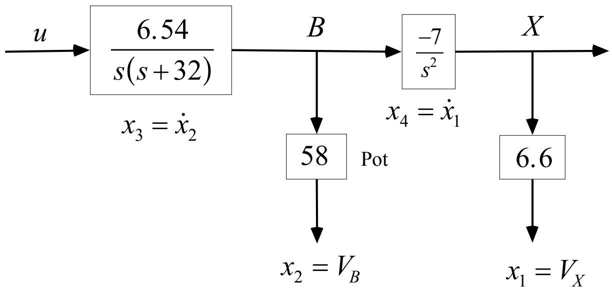


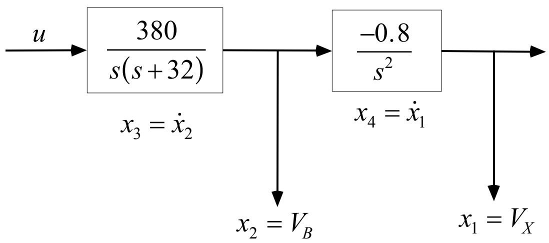


Experimental data give

$$\begin{cases} \tau = 0.032 \text{ sec} \\ K_B = 57.95\ V/rad\ \ (Pot), \quad K_X = 6.6\ V/m \end{cases}$$

$$\begin{cases} KK_B = 10.444 \sim 13.833 \\ K_T = \dfrac{B_{dot}}{B_{tacho}} = -2.44 \sim -2.49 \end{cases}$$





$$x_3 = \dot{x}_2 = K_T B_{Tacho}, \qquad K_T = \frac{B_{dot}}{B_{tacho}} = -2.44 \sim -2.49$$

$$\begin{cases} \dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u \\ \mathbf{y} = \mathbf{C}\mathbf{x} + \mathbf{D}u \end{cases}$$

$$\mathbf{A} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -32 & 0 \\ 0 & -0.8 & 0 & 0 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 0 \\ 0 \\ 380 \\ 0 \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, \quad \mathbf{D} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

For a reasonable control effort, choose system eigenvalues as

$$\mathbf{P} = [-5, \quad -5, \quad -5, \quad -5]$$

so a control gain is

$$\mathbf{K} = [-2.056, \quad 0.395, \quad -0.032 \quad -1.645]$$

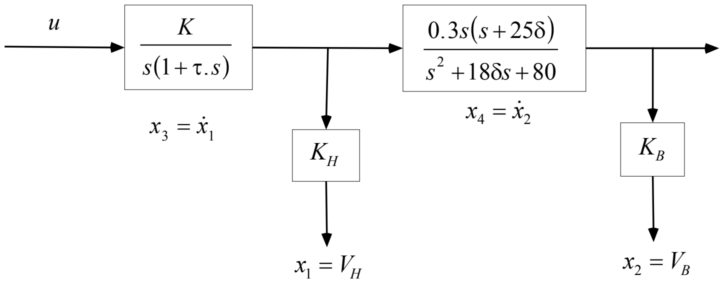
and an estimator is

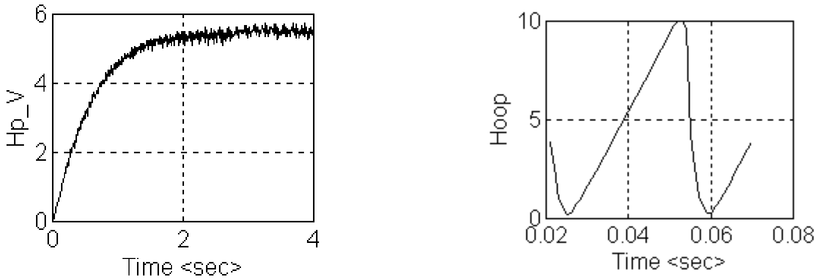
$$\mathbf{A}_e = \begin{bmatrix} -25, & 0, & 0, & 1 \\ 0, & -25, & 1, & 0 \\ 0, & 0, & -32, & 0 \\ 0, & 0, & 0, & 0 \end{bmatrix}, \quad \mathbf{L} = \begin{bmatrix} 25, & 0 \\ 0, & 25 \\ 0, & 0 \\ 0, & -0.8 \end{bmatrix}$$

Remark: The allocation of 4 identical eigenvalues is similar to that of ITAE and Bessel filter, but it simplifies the controllers as above.

Ball-Hoop System

Duy-Ky Nguyen **13-11-1993**



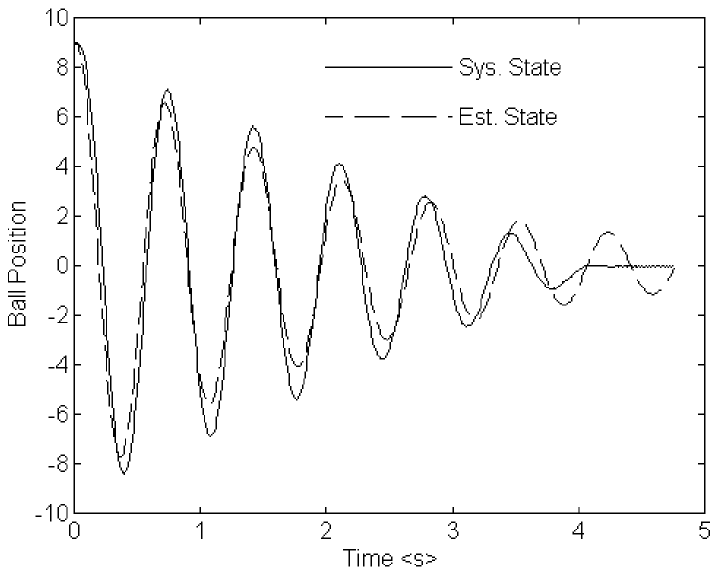


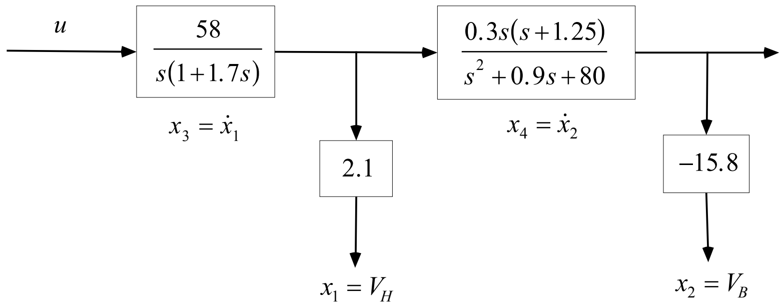
Experimental data give

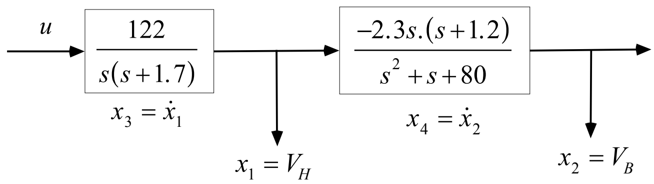
$$\begin{cases} \tau = 0.568 \sim 0.6 \text{ sec} \\ K_H = 2.1\ V/rad, \quad K_B = -15.8\ V/rad \end{cases}$$

$$\begin{cases} KK_H = 65.487 \sim 77.290 \\ K_T = \dfrac{H_{dot}}{H_{tacho}} = 65.62 \sim 66.80 \end{cases}$$

With
$\delta = 0.05$
we obtain







$$x_3 = \dot{x}_1 = K_T H_{Tacho}, \qquad K_T = \frac{H_{dot}}{H_{tacho}} = 66$$

$$\begin{cases} \dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u \\ \mathbf{y} = \mathbf{C}\mathbf{x} + \mathbf{D}u \end{cases}$$

where

$$\mathbf{A} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1.7 & 0 \\ 0 & -80 & 1.2 & -1 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 0 \\ 0 \\ 122 \\ -281 \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, \quad \mathbf{D} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

For a reasonable control effort, choose system eigenvalues as

$$\mathbf{P} = \begin{bmatrix} -12, & -12, & -12, & -12 \end{bmatrix}$$

so a control gain is

$$\mathbf{K} = \begin{bmatrix} 2.125, & -1.720, & 0.609, & 0.103 \end{bmatrix}$$

and an estimator is

$$\mathbf{A}_e = \begin{bmatrix} -60, & 0, & 1, & 0 \\ 0, & -60, & 0, & 1 \\ 0, & 0, & -1.7, & 0 \\ 0, & 0, & 1.2, & -1 \end{bmatrix}, \quad \mathbf{L} = \begin{bmatrix} 60, & 0 \\ 0, & 60 \\ 0, & 0 \\ 0, & -80 \end{bmatrix}$$

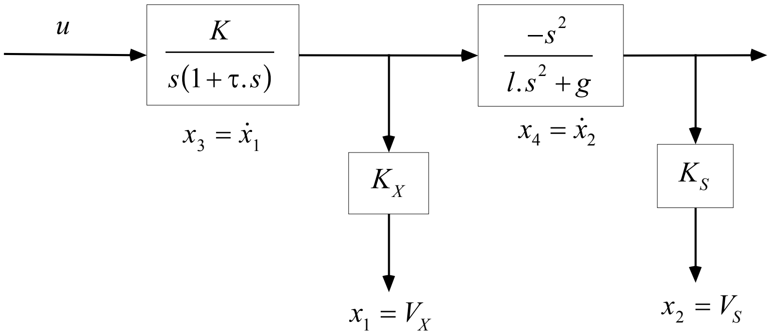
Remark: The 4 identical eigenvalues are the best, this allocation is much better than that of ITAE and Bessel filter. Particularly, with 2 identical eigenvalues, there are identical pole and zero for elimination in each controller and we have come up with the simple controllers above.

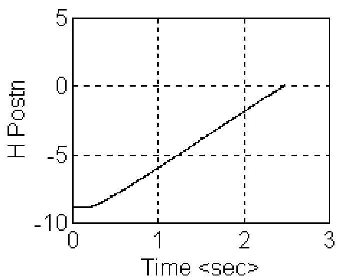
Problem: there is a common factor of s in the feedforward path, so the step response is of *zero*. It is wrong with the experimental results.

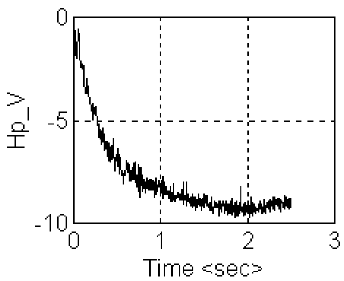
OverHead-Crane System

Duy-Ky Nguyen

29-05-1994







Experimental data give

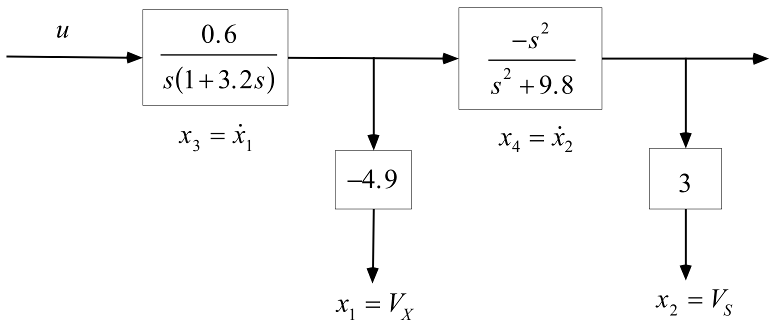
$$\begin{cases} \tau = 0.305 \sim 0.32 \text{ sec} \\ K_X = -4.9\ V/m, \quad K_S = 3\ V/rad \end{cases}$$

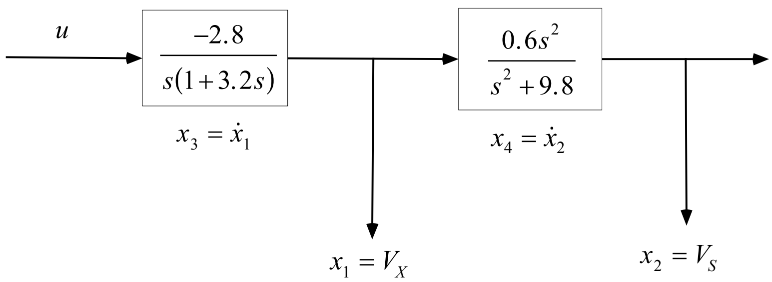
$$\begin{cases} KK_X = -0.862 \sim -0.910 \\ K_T = \dfrac{H_{dot}}{H_{tacho}} = -0.438 \sim -0.46 \end{cases}$$

With

$$l = 1\ m, \quad g = 9.8\ ms^{-2}$$

we obtain





$$x_3 = \dot{x}_1 = K_T X_{Tacho}, \qquad K_T = \frac{X_{dot}}{X_{tacho}} = -0.46$$

$$\begin{cases} \dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u \\ \mathbf{y} = \mathbf{C}\mathbf{x} + \mathbf{D}u \end{cases}$$

where

$$\mathbf{A} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -3.2 & 0 \\ 0 & -9.8 & 1.92 & -1 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 0 \\ 0 \\ -2.8 \\ -1.68 \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, \quad \mathbf{D} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

For a reasonable control effort, choose system eigenvalues as

$$\mathbf{P} = \begin{bmatrix} -2.5, & -2.5, & -2.5, & -2.5 \end{bmatrix}$$

so a control gain is

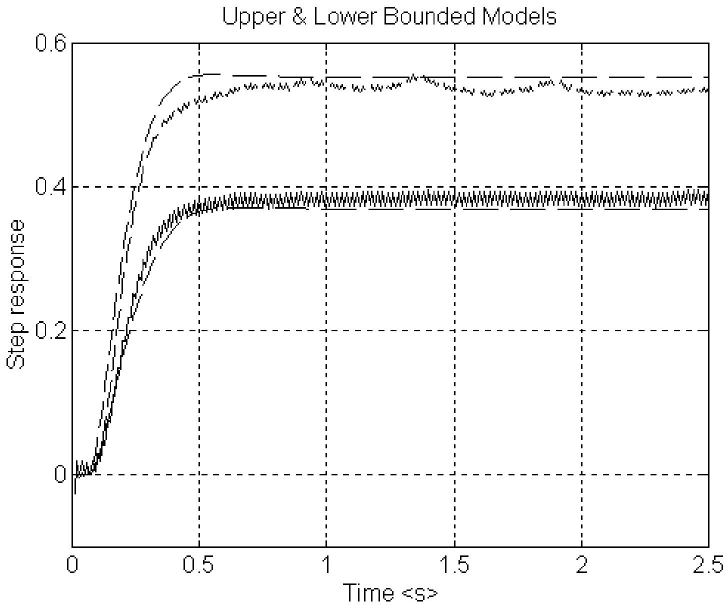
$$\mathbf{K} = \begin{bmatrix} -1.424, & -14.115 & -1.135 & -2.156 \end{bmatrix}$$

and an estimator is

$$\mathbf{A}_e = \begin{bmatrix} -12.5, & 0, & 1, & 0 \\ 0, & -12.5, & 0, & 1 \\ 0, & 0, & -3.2, & 0 \\ 0, & 0, & -1.92, & 0 \end{bmatrix}, \quad \mathbf{L} = \begin{bmatrix} 12.5, & 0 \\ 0, & 12.5 \\ 0, & 0 \\ 0, & -9.8 \end{bmatrix}$$

Static-VAR System

Duy-Ky Nguyen **5-11-1994**



From the figure above we can estimate the system model by the following upper & lower bounded models by different step responses at different torque loads

$$G_{p1}(s) = \frac{37.e^{-s.T_d}}{s^2 + 17s + 100}, \qquad G_{p2}(s) = \frac{83.e^{-s.T_d}}{s^2 + 21s + 150}$$

and a nominal model can be chosen as

$$G_p(s) = \frac{60.e^{-s.T_d}}{s^2 + 19s + 125}$$

where

$T_d = 0.05$ sec : transportation lag

If the sampling time is taken into account, a digital-equivalent model can be

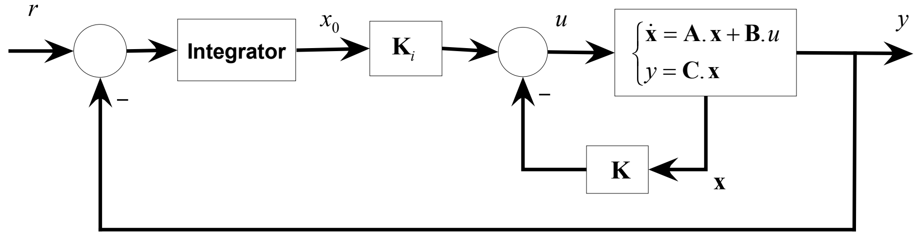
$$G_p(s) \approx \frac{60.\left(1 - T_d.s + \frac{T_d^2}{2}s^2\right)}{\left(s^2 + 19s + 125\right)\left(\frac{T_s}{2}s + 1\right)}$$

if the sampling-time is ignored

$$G_p(s) \approx \frac{60.(1 - T_d.s)}{s^2 + 19s + 125}$$

this model will be used in designing a controller.

Remark. the reference will be the step of 5 since the voltage 5 volts is equivalent to the unity power factor.



Let

$$x_0(t) = \int_{t_0}^{t} e(\tau).d\tau, \quad e(t) = y(t) - r(t)$$

be an integral of the error between the output and the reference input for the desired output, so

$$\dot{x}_0 = e = y - r = \mathbf{C}\mathbf{x} - r$$

then

$$\dot{\tilde{\mathbf{x}}} = \mathbf{A}_i.\tilde{\mathbf{x}} + \mathbf{B}_i.u - \mathbf{r}$$

where

$$\tilde{\mathbf{x}} = \begin{bmatrix} x_0 \\ \mathbf{x} \end{bmatrix}, \quad \mathbf{A}_i = \begin{bmatrix} 0 & \mathbf{C} \\ 0 & \mathbf{A} \end{bmatrix}, \quad \mathbf{B}_i = \begin{bmatrix} 0 \\ \mathbf{B} \end{bmatrix}, \quad \mathbf{r} = \begin{bmatrix} r \\ \mathbf{0} \end{bmatrix}$$

$$\mathbf{A}_i = \begin{bmatrix} 0 & 1 & -0.05 \\ 0 & 0 & 1 \\ 0 & -125 & -19 \end{bmatrix}, \quad \mathbf{B}_i = \begin{bmatrix} 0 \\ 0 \\ 60 \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} 0 & 1 & -0.05 \end{bmatrix}, \quad \mathbf{r} = \begin{bmatrix} 5 \\ 0 \\ 0 \end{bmatrix}$$

choose the poles in s-domain at -8, we obtain the following gain matrix

$$\mathbf{K} = \begin{bmatrix} 8.5333 & 1.5433 & 0.0833 \end{bmatrix}$$

For an integral state-space control, we have used

$$u = -\mathbf{K}.\begin{bmatrix} \int (y - r).dt \\ y \\ \hat{x}_2 \end{bmatrix}, \quad \mathbf{K} = \begin{bmatrix} 8.533, & 1.117 & 0.083 \end{bmatrix}$$

where the estimate state $\hat{x}_2$ is deternined by the following state-space observer

$$\dot{\hat{\mathbf{x}}} = \mathbf{A}_e.\hat{\mathbf{x}} + \mathbf{B}.u + \mathbf{L}.y, \quad \hat{\mathbf{x}} = \begin{bmatrix} \hat{x}_1 & \hat{x}_2 \end{bmatrix}^T$$

with

$$\mathbf{B} = \begin{bmatrix} 0 \\ 60 \end{bmatrix}, \quad \mathbf{L} = \begin{bmatrix} 61 \\ 316 \end{bmatrix} \Rightarrow \mathbf{A}_e = \mathbf{A} - \mathbf{LC} = \begin{bmatrix} -61, & 1 \\ -441, & -19 \end{bmatrix}$$