PD Controller and Practical System Identification for High Order Systems

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1. Problem Statements

Error and its change are used in a fuzzy controller as in a PD controller, $G_c(s) = K_p + K_d s$. These 2 gains are determined on the basis of dynamic closed-loop requirements by solving 2 equations of unknowns $K_p \& K_d$. These 2 equations come from the characteristic equation computed at 2 Hurwitz real roots or 1 Hurwitz complex root which produces 2 equations with respect to the real and imaginary parts, since the conjugate root is also Hurwitz for a real coefficient characteristic equation. For a second order system, these 2 Hurwitz roots do guarantee a stable closed-loop system since the characteristic equation has only 2 roots. For a higher *n*-order system, the characteristic equation has *n* roots, so there is no guarantee for other $(n-2)$ roots to be Hurwitz; thus the root-locus technique is required to analyze the existence of a stabilizing PD controller and to design if any.

The above discussion is consistent with the state-space control theory in the fact that there is a restricted pole-placement for the desired closed-loop poles if a full-state feedback is not used. For a second order system, a full-state feedback is used so there is no restriction as in a PD controller. So a state-space controller can be considered as a PD controller when all the gains of x_3 and above are equal to *zero*.

Consider the following *n*-th order system

$$G_p(s) = \frac{b_{n-1}s^{n-1} + \cdots + b_1 s + b_0}{s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0} \tag{1}$$

and its state-space model

$$\begin{cases} \dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u \\ y = \mathbf{C}\mathbf{x} \end{cases} \tag{1}$$

where

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & 0 \\ 0 & 0 & \cdots & 0 & 1 \\ -a_0 & -a_1 & \cdots & -a_{n-2} & -a_{n-1} \end{bmatrix}, \mathbf{B} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}, \mathbf{C} = \begin{bmatrix} b_0 & b_1 & \cdots & b_{n-2} & b_{n-1} \end{bmatrix}$$

and

$$u = -\mathbf{K}\mathbf{x} \tag{2}$$

then

$$\dot{\mathbf{x}} = (\mathbf{A} - \mathbf{B}\mathbf{K})\mathbf{x} \Rightarrow s\mathbf{X} = (\mathbf{A} - \mathbf{B}\mathbf{K})\mathbf{X} \Rightarrow (s\mathbf{I} - \mathbf{A} + \mathbf{B}\mathbf{K})\mathbf{X} = \mathbf{0} \tag{3}$$

thus the characteristic equation is

$$|s\mathbf{I} - \mathbf{A} + \mathbf{B}\mathbf{K}| = s^n + (k_n + a_{n-1})s^{n-1} + \cdots + (k_3 + a_2)s^2 + (k_2 + a_1)s + (k_1 + a_0) = 0 \tag{4}$$

For a state-space controller to be considered as a PD controller, that is only the first 2 states are used in a feedback, let

$$k_i = 0, i = 3, \cdots, n \tag{5}$$

then the desired closed-loop roots must be chosen to satisfy the following equation

$$s^n + a_{n-1}s^{n-1} + \cdots + a_2 s^2 + (k_2 + a_1)s + (k_1 + a_0) = 0 \tag{6}$$

Since there is no analytical solution for a higher order equation, we rewrite Eq.(7) in the following form

$$\frac{k_1 + k_2 s}{s^n + a_{n-1}s^{n-1} + \cdots + a_2 s^2 + a_1 s + a_0} = -1 \tag{7}$$

then the root-locus technique can be applied for a stabilizing reduced-order state-space controller. Note that this form is of a PD controller.

As we have seen that it is always possible to design a PD controller or a state-space controller with the first 2 states (error and its derivative) for a second order system, so is it for a fuzzy controller; however the root-locus technique has been used to analyze the existence of such controller for a higher order system and to design if any. We will consider the existence of a stabilizing controller for a higher order system, and if it exists we will find a second order model that can estimate that higher order system on the basis of step response so this can facilitate to identify a system as a second order model in a practical system identification.

2. Third Order Systems

2.1. $G_p(s) = \frac{1}{s^3}$

The root-locus in Fig.1.*a* shows that there is *no* PD controller for this system.

2.2. $G_p(s) = \frac{1}{s^2(s+a)}$

The root-locus in Fig.1.*b* shows that there exists a PD controller to stabilize the system, however the desired root cannot be to the left of $s = -a/2$.

For example, the system $G_p(s) = \frac{10}{s^2}$ can be an approximate second order system of the system $G_p(s) = \frac{1}{s^2(s+1)}$ can be estimated as in Fig.1.*c*. This approx second order system is used to determine $u = -\mathbf{K}[x_1 \quad x_2]^T$ for the third order system as in Fig.1.*d*.

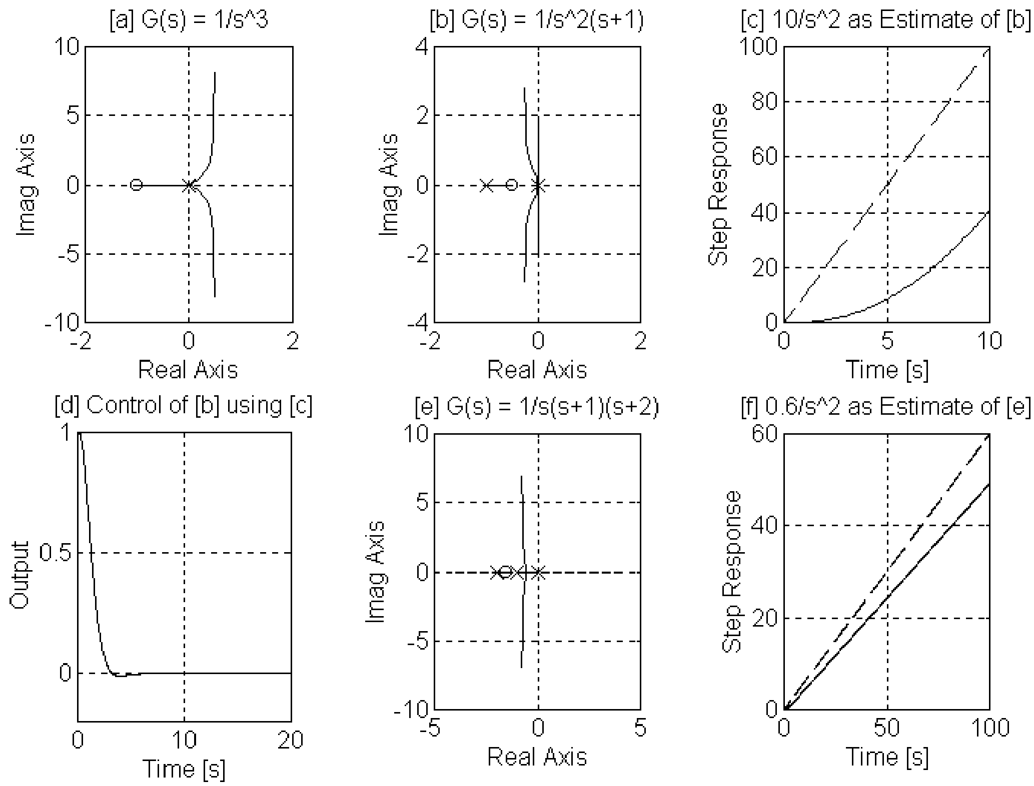

Fig.1:

2.3. $G_p(s) = \dfrac{1}{s(s+a)(s+b)}$

The root-locus in Fig.1.*e* shows that there exists a PD controller to stabilize the system.

For example, the system $G_p(s) = \dfrac{0.6}{s^2}$ can be an approximate second order system of the system $G_p(s) = \dfrac{1}{s(s+1)(s+2)}$ can be estimated as in Fig.1.*f*. This approx second order system is used to determine $u = -\mathbf{K}[x_1 \quad x_2]^T$ for the third order system as in Fig.2.*a*.

2.4. $G_p(s) = \dfrac{1}{(s+a)(s+b)(s+c)}$

The root-locus in Fig.2.*b* shows that there exists a PD controller to stabilize the system.

For example, the system $G_p(s) = \dfrac{0.5}{(s+1)(s+3)}$ can be an approximate second order system of the system $G_p(s) = \dfrac{1}{(s+1)(s+2)(s+3)}$ can be estimated as in Fig.2.*c*. This approx second order system is used to determine $u = -\mathbf{K}[x_1 \quad x_2]^T$ for the third order system as in Fig.2.*d*.

3. Fourth Order Systems

3.1. $G_p(s) = \dfrac{1}{s^3(s+a)}$

The root-locus in Fig.2.*e* shows that there is *no* PD controller for this system.

3.2. $G_p(s) = \dfrac{1}{s^2(s+a)(s+b)}$

The root-locus in Fig.2.*f* shows that there is *no* PD controller for this system.

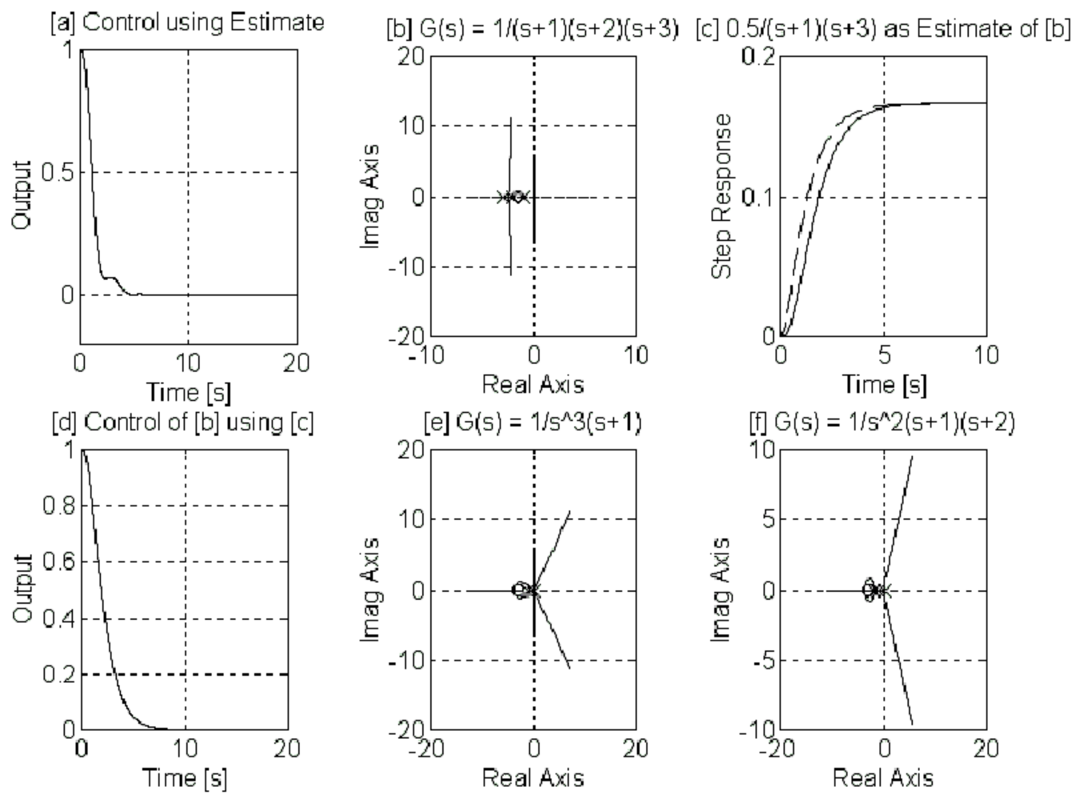

Fig.2:

3.3. $G_p(s) = \dfrac{1}{s(s+a)(s+b)(s+c)}$

The root-locus in Fig.3.*a* shows that there exists a PD controller to stabilize the system.

For example, the system $G_p(s) = \dfrac{0.4}{s^2}$ can be an approximate second order system of the system $G_p(s) = \dfrac{1}{s(s+1)(s+2)(s+3)}$ can be estimated as in Fig.3.*b*. This approx second order system is used to determine $u = -\mathbf{K}[x_1 \quad x_2]^T$ for the fourth order system as in Fig.3.*c*.

3.4. $G_p(s) = \dfrac{1}{(s+a)(s+b)(s+c)(s+d)}$

The root-locus in Fig.3.*d* shows that there exists a PD controller to stabilize the system.

For example, the system $G_p(s) = \dfrac{0.04}{(s+1)(s+3)}$ can be an approximate second order system of the system $G_p(s) = \dfrac{1}{(s+1)(s+2)(s+3)(s+4)}$ can be estimated as in Fig.3.*e*. This approx second order system is used to determine $u = -\mathbf{K}[x_1 \quad x_2]^T$ for the fourth order system as in Fig.3.*f*.

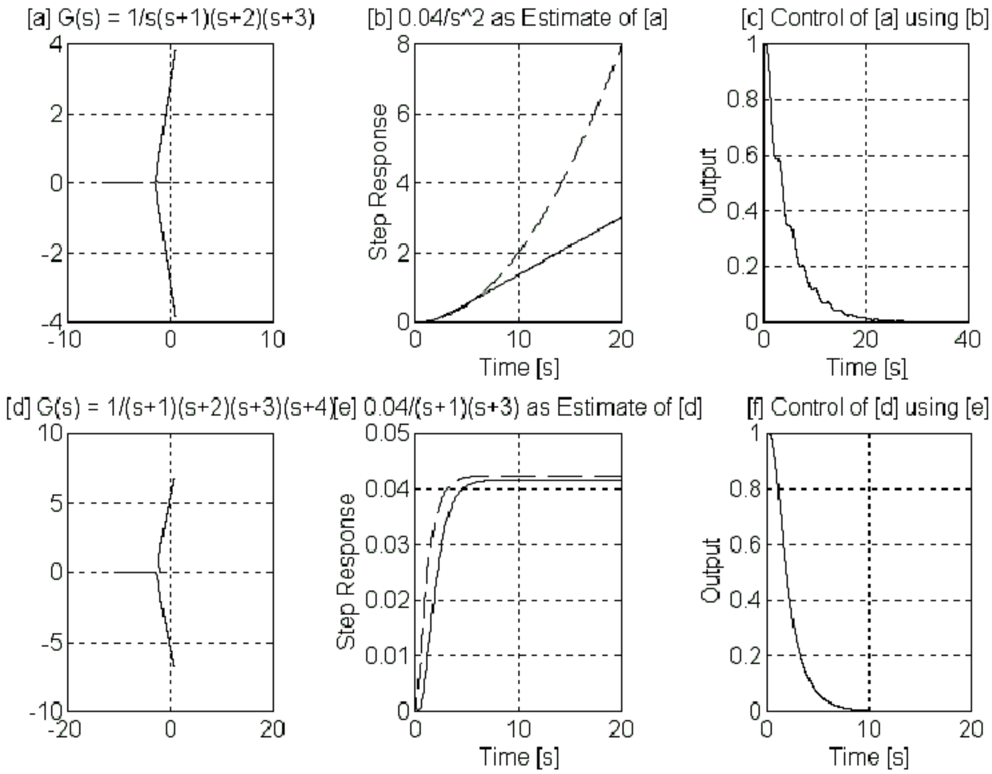

Fig.3:

4. Conclusion

A PD controller can be always designed with any desired roots in LHP for a second order system, so is for a fuzzy controller. For a higher order system, the root-locus technique has been used to analyze the existence of a stabilizing controller, and if it exist we can use an approximate second order using step responses to design it using the desired closed-loop poles based on the open-loop system dynamics. This can be seen as the desired closed-loop poles are chosen in some restricted region in LHP.

In the real world, most dynamic systems can be considered as composing of several second order subsystems and each PD controller, and hence each fuzzy controller, can be designed for each